Linearization of Transmitter and Receiver Nonlinearities in Optical OFDM Transmission

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I. Introduction

ORTHOGONAL Frequency Division Multiplexing (OFDM) has recently (e.g., [1]) been proposed for optical long-haul high-speed data transmission over single-mode fibers due to the simplicity of equalization of the dispersion dominated optical channel. However, the modulator and detector components of the optical transmission system exhibit severe nonlinearities [2] which pose a challenge to OFDM. The impact of these nonlinear effects has been presented in a previous work [3], where also the system performance's dependence on the setup parameters has been analyzed.

In this paper, linearization of transmitter and receiver nonlinearities using predistortion functions is presented and analyzed, then the bit error performance of a linearized system in dependence on the setup parameters is simulated.

II. System model

The Zero-IF (Intermediate Frequency) Intensity Modulation/Direct Detection (IM/DD) system considered in this paper is able to transmit real valued signals only, thus, the OFDM signal has to be crafted in a special manner to ensure that this requirement is fulfilled, e.g. by complex conjugate extension of subcarriers. For the theoretical considerations made here, the OFDM signal is modelled by a real valued, zero mean process $x(t)$ with gaussian probability density function and variance σ_x^2.

The overall nonlinear hardware characteristic of an IM/DD system in back-to-back operation (i.e. with no channel involved) can be described by a function

$$r(t) = \beta^2 \cos^2(g^0_{-\pi/2}(m \cdot s(t) + u_{\text{bias}})), \tag{1}$$

that maps a modulation signal $s(t)$ onto a detector output signal $r(t)$. In this characteristic, $g^0_{-\pi/2}(\cdot)$ represents a hard clipping characteristic with fixed clipping thresholds $\vartheta_{\text{low}} = -\pi/2$ and $\vartheta_{\text{high}} = 0$, while m and u_{bias} are variable setup parameters and β is an arbitrary power scaling factor which is set to $\beta = 1$ for the considerations made in this abstract.

This nonlinearity is supposed to be linearized employing functions $\varsigma(\xi)$ at the transmitter side and $\nu(\rho)$ at the receiving side, resulting in an overall system with output

$$z(t) = \nu(\beta^2 \cos^2(g^0_{-\pi/2}(m \cdot \varsigma(x(t)) + u_{\text{bias}}))), \tag{2}$$

that is linear in a certain operating range.

The receiver output signal $z(t)$ again is a stochastic process which generally is neither zero-mean, nor follows a gaussian distribution.

A. Linearization Approach

If predistortion is performed by use of an arc cosine expression

$$\varsigma(\xi) = \frac{1}{m}\left(-\arccos(m_{\text{pre}} \cdot \xi + b_{\text{pre}}) - u_{\text{bias}}\right) \tag{3}$$

similar to the arc sine function proposed in [5] for coherent systems, and at the receiver side the square root of the detector output signal $r(t)$ is taken, i.e.

$$z(t) = \nu(r(t)) = \sqrt{r(t)}, \tag{4}$$

the resulting system can be described by an equivalent system

$$z(t) = \beta g^1_0(m_{\text{pre}} \cdot x(t) + b_{\text{pre}}), \tag{5}$$

with $g^1_0(\cdot)$ being a hard clipping function with lower threshold 0 and upper threshold 1.

Here m_{pre} and b_{pre} are user-selectable parameters, similar to m and u_{bias} in (1), but with the latter being hardware system setup parameters representing physical properties and the former being parameters only applied in digital signal preprocessing.

There is, however, a connection between these setup parameters. Defining the physical modulator input signal u_{mod} as

$$u_{\text{mod}} = m \cdot s(t) + u_{\text{bias}}, \tag{6}$$

this expression evaluates for non-linearized case to $u_{\text{mod}} = m \cdot x(t) + u_{\text{bias}}$, while in the linearized case, the modulator input signal equals

$$u_{\text{mod}} = -\arccos(m_{\text{pre}} \cdot x(t) + b_{\text{pre}}), \tag{7}$$

which that a predistortion bias b_{pre} will cause a physical bias

$$u_{\text{bias}} = -\arccos b_{\text{pre}} \tag{8}$$

which can be found by Taylor series expansion of (7) around $x(t) = 0$. The linear term of this expansion represents the equivalent of the driving level m and evaluates to

$$m = \frac{m_{\text{pre}}}{\sqrt{1 - b^2_{\text{pre}}}}. \tag{9}$$

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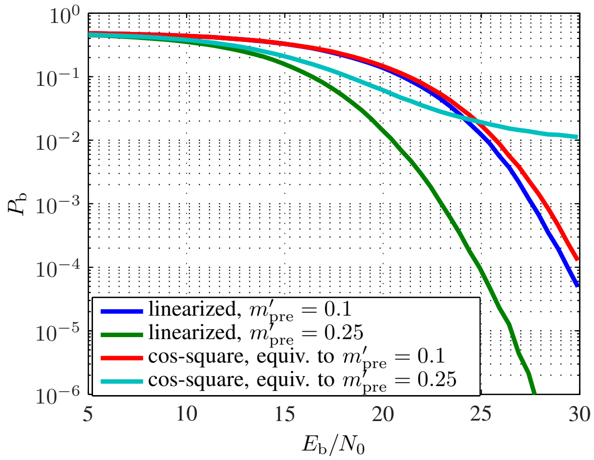

Fig. 1. Comparison of bit error rates P_b for linearized and non-linearized system in dependence on the per-bit SNR E_b/N_0

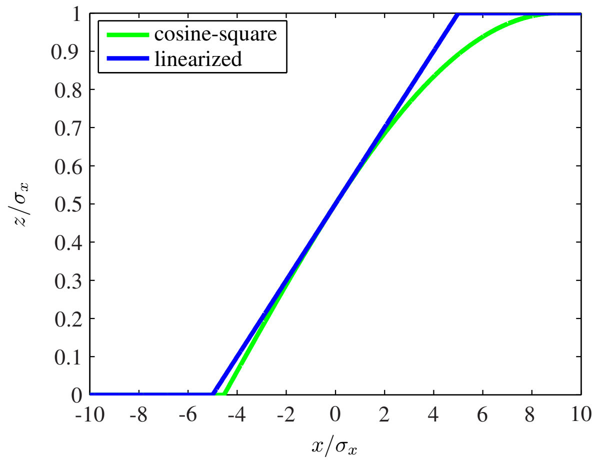

Fig. 2. Comparison of characteristics of linearized and non-linearized system for $b_\mathrm{pre} = 0.5$ and $m'_\mathrm{pre} = 0.1$

III. System Performance

Following the approach pursued in [3], the system performance of the linearized system depends on the system setup parameters m_pre and b_pre.
Modelling the output

$$z(t) = \beta g_{0,1}(m_\mathrm{pre} \cdot x(t) + b_\mathrm{pre}), \quad (10)$$

of the linearized system as a scaled version of $x(t)$ with an additional, uncorrelated distortion term $d(t)$ [4]

$$z(t) = \alpha_\mathrm{lin} \cdot x(t) + d(t), \quad (11)$$

the scaling factor evaluates to

$$\alpha_\mathrm{lin} = \beta \cdot \frac{m_\mathrm{pre}\sigma_x}{2} \left(\mathrm{erf}\Big(\frac{1 - b_\mathrm{pre}}{\sqrt{2} m_\mathrm{pre}\sigma_x}\Big) - \mathrm{erf}\Big(\frac{-b_\mathrm{pre}}{\sqrt{2} m_\mathrm{pre}\sigma_x}\Big) \right), \quad (12)$$

where $\mathrm{erf}(\cdot)$ is the error function.
Since $\mathrm{E}\left\{z(t)^2\right\}$ can be calculated analytically, the power of the interference $\mathrm{E}\left\{d(t))^2\right\}$ thus is known, which allows for the analysis of the overall Signal-to-Interference+Noise-Ratio in dependence on the system parameters, which will be performed in the final paper.

IV. Simulation Results

The bit error performance of a linearized IM/DD optical OFDM system with 1024 subcarriers employing QPSK modulation has been evaluated by means of Monte Carlo simulations. DC and Nyquist frequency subcarrier have been set to zero, 511 information bearing subcarriers have been extended in conjugate complex fashion to ensure a real valued modulation signal. Since here only the back-to-back case is considered, the length of the cyclic prefix has been set to zero. Fig. 1 shows the simulated bit error rates for such a linearized system in comparison to a non-linearized system with cosine-square characteristic as analyzed in [3]. The setup parameters of the linearized system have been fixed to $b_\mathrm{pre} = 0.5$ and $m'_\mathrm{pre} = m_\mathrm{pre}\sigma_x = \{0.1, 0.25\}$, and to ensure a fair comparison, the hardware system setup parameters u_bias and m were chosen according to (8) and (9). It can be seen that the linearized and non-linearized system perform similarly for small driving levels m'_pre, which can be explained by the fact that in a small region around the operation point the characteristics do not deviate significantly from each other, as Fig. 2 shows. If m'_pre is increased to 0.25, the performance of the linearized system improves due to the increase of α_lin, but the non-linearized system is limited by interference introduced by its cosine-square characteristic.
In our final contribution, results of further bit error simulations will be presented, pointing out the dependence of the system performance on the setup parameters in particular.

V. Conclusion

In our paper, an approach for linearization of the cosine-square overall characteristic of an IM/DD optical OFDM system is presented and the system performance is estimated by means of statistical measures of the system's output process. In our final submission, we will explain the required derivations in more detail and also introduce an analysis of a power constrained transmission system, as most practical systems are. Bit error simulations are performed for the general as well as the power constrained case, and their results will be compared to those for the non-linearized system.

References

[1] A.J. Lowery, Liang Du, J. Armstrong, *Orthogonal Frequency Division Multiplexing for Adaptive Dispersion Compensation in Long Haul WDM Systems*, Optical Fiber Communication Conference (OFC), PDP39, March 2006.

[2] Govind P. Agrawal, *Fiber-Optic Communication Systems, Third Edition*, John Wiley & Sons, Inc., 2002.

[3] Henning Paul and Karl-Dirk Kammeyer, *Modeling and Influences of Transmitter and Receiver Nonlinearities in Optical OFDM Transmission*, Proc. of the 13th International OFDM Workshop 2008 (InOWo '08), Hamburg, Germany, Aug. 2008.

[4] Julian J. Bussgang, *Crosscorrelation Functions of Amplitude-Distorted Gaussian Signals*, PhD thesis, Massachusetts Institute of Technology, 1952.

[5] Yan Tang, Keang-Po Ho and William Shieh, *Coherent Optical OFDM Transmitter Design Employing Predistortion*, IEEE Photonics Technology Letters, Vol. 20, No. 11, June 1st, 2008.