Efficient Near Maximum-Likelihood Decoding of Multistratum Space-Time Codes

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***Abstract*— Multistratum space-time codes combine the layered transmission strategy of V-BLAST with space-time block coding in order to achieve high data rates and at the same time exploit transmit diversity. Simple detection schemes like successive interference cancellation severely suffer from the strong correlation between the strata, so true maximum-likelihood detection needs to be performed instead, which can be efficiently realized by the sphere decoding algorithm. However, most implementations known from the literature still have relatively high complexity, especially for low signal to noise ratio. Therefore, we propose several modifications that can reduce the computational effort by orders of magnitude while achieving at least near optimum performance.**

***Index Terms*— V-BLAST, Space-Time Codes, Sphere Decoding.**

I. Introduction

Multiple antenna systems offer large capacities in rich scattering environments. Multilayer concepts like V-BLAST achieve very high data rates with reasonable decoding complexity [1]. On the other hand, the transmission quality may be significantly improved by appropriate space-time coding [2]. Multistratum space-time codes (MSSTC) represent a combination of these two approaches. Here, the parallel data streams (called strata) experience full transmit diversity due to the application of a space-time block code [3].

The strata usually strongly interfere even for uncorrelated channels. As a consequence, successive interference (SIC), which is commonly used for the detection of V-BLAST, yields extremely poor performance. Brute force maximum-likelihood (ML) estimation is usually prohibitively complex, but the effort can be significantly reduced by the sphere decoding (SD) algorithm [4]. However, for multistratum codes it is still quite high due to the large number of parameters.

In this paper, a new quasi-orthogonal multistratum space-time code that achieves full rate with full diversity is presented. Furthermore, we propose various enhancements for the standard SD algorithm that achieve (close to) optimum performance with a computational complexity comparable to that of simple SIC. The first two modifications are based on the efficient detection algorithm from [5]. The third one can be interpreted as adaptive tree pruning. In contrast to existing pruning strategies, the proposed method does not require restarts as it always finds at least one feasible point.

II. Notation

Throughout this paper, we use bold lower (upper) case letters for column vectors (matrices) and an underline to indicate complex quantities. The superscript $\cdot^T$ denotes transpose, $\cdot^*$ complex conjugation, $\otimes$ represents the Kronecker product, and $\mathcal{Q}\{\cdot\}$ means quantization to the nearest symbol of the modulation alphabet. Finally, for an arbitrary complex $m \times n$ matrix $\underline{\mathbf{M}}$ we define the corresponding real $2m \times n$ matrix $\mathbf{M}$, that alternately contains the real and imaginary part of each row.

III. System Model

We consider a single-user MIMO system with N_T transmit and N_R receive antennas in a flat fading environment. The $N_R \times N_T$ channel matrix $\underline{\mathbf{H}}$ is assumed to contain independent complex Gaussian entries. The $N_R \times 1$ receive vector is given by

$$\underline{\mathbf{y}}[k] = \underline{\mathbf{H}}\,\underline{\mathbf{x}}[k] + \underline{\mathbf{n}}[k]\;, \tag{1}$$

where $\underline{\mathbf{x}}[k]$ contains all transmit symbols and $\underline{\mathbf{n}}[k]$ represents complex white Gaussian noise of variance $\sigma^2_{\underline{n}}$.

In general, a space-time codeword consists of N_{ST} consecutive transmit vectors and contains K_{ST} information bearing symbols $\underline{\mathbf{d}} = (\underline{d}_1, \ldots, \underline{d}_{K_{ST}})^T$. In order to allow for the transmission of conjugate complex information symbols, it is necessary to split up $\underline{\mathbf{d}}$ into real and imaginary part as described in Section II, because complex conjugation is not a linear operation. With the resulting $2K_{ST} \times 1$ vector $\mathbf{d}$, one codeword can be expressed as

$$\underline{\mathbf{x}} = \left(\underline{\mathbf{x}}^T[1], \ldots, \underline{\mathbf{x}}^T[N_{ST}]\right)^T = \underline{\mathbf{G}}\,\mathbf{d} \tag{2}$$

where the generator matrix $\underline{\mathbf{G}}$ of dimension $N_{ST}N_T \times 2K_{ST}$ defines the structure of the employed space-time code. The corresponding observations at the receiver are summarized in the vector

$$\begin{aligned} \underline{\mathbf{y}} &= \left(\underline{\mathbf{y}}^T[1], \ldots, \underline{\mathbf{y}}^T[N_{ST}]\right)^T \\ &= (\mathbf{I}_{N_{ST}} \otimes \underline{\mathbf{H}})\,\underline{\mathbf{x}} + \underline{\mathbf{n}} = (\mathbf{I}_{N_{ST}} \otimes \underline{\mathbf{H}})\,\underline{\mathbf{G}}\,\mathbf{d} + \underline{\mathbf{n}} \\ &= \underline{\mathbf{A}}\,\mathbf{d} + \underline{\mathbf{n}}\;. \end{aligned} \tag{3}$$

In (3), the $N_{ST}N_R \times 2K_{ST}$ system matrix $\underline{\mathbf{A}}$ describing the joint effects of space-time coding and mobile radio channel was introduced. As the symbols in $\mathbf{d}$ that need to be estimated at the receiver are real, we also partition the complex matrices and vectors in (3) into their real and imaginary parts and arrive at the real-valued linear system model

$$\mathbf{y} = \mathbf{A}\,\mathbf{d} + \mathbf{n}, \tag{4}$$

which will be used exclusively throughout this paper. Note that with the normalization $\mathrm{E}\left\{\mathrm{tr}\left\{\mathbf{A}^T\mathbf{A}\right\}\right\} = 2K_{ST}$, the average signal to noise ratio at the receiver when using M-QAM is

$$\frac{\sigma_d^2}{\sigma_n^2} = \frac{\sigma_{\underline{d}}^2}{\sigma_{\underline{n}}^2} = \log_2(M)\frac{E_b}{N_0}, \tag{5}$$

where E_b denotes the mean energy per information bit and N_0 is the one-sided spectral power density of the noise.

IV. Transmission Strategies

In this section we briefly review the V-BLAST architecture and space-time block codes. Afterwards, multistratum space-time codes will turn out to be a combination of these two concepts.

A. V-BLAST

Information theoretical results show that in rich fading environments the channel capacity of MIMO systems grows almost linearly with the number of transmit antennas as long as $N_T \leq N_R$. A convenient way to realize high data rates is based on the simultaneous transmission of parallel data streams. For V-BLAST, the signals at different transmit antennas are independent of each other [1]. Consequently, the $N_T \times 2N_T$ generator matrix has block diagonal structure,

$$\underline{\mathbf{G}} = \mathbf{I}_{N_T} \otimes \begin{pmatrix} 1 & j \end{pmatrix}, \tag{6}$$

and it is possible to transmit N_T symbols per channel use. However, V-BLAST does not take advantage of transmit diversity in contrast to the space-time codes described next.

B. Space-Time Block Codes

Indoor scenarios typically offer no or only little temporal and frequency diversity. However, in a rich scattering environment it is possible to exploit spatial diversity in order to improve transmission quality. While receive diversity can easily be gained by maximum ratio combining, transmit diversity requires appropriate space-time coding [2].

Orthogonal space-time block codes are an attractive approach because of their very low decoding complexity. A simple example is the well-known Alamouti scheme [6] for $N_T = 2$ transmit antennas

$$\underline{\mathbf{X}} = \begin{pmatrix} \underline{\mathbf{x}}[1] & \underline{\mathbf{x}}[2] \end{pmatrix} = \begin{pmatrix} \underline{d}_1 & -\underline{d}_2^* \\ \underline{d}_2 & \underline{d}_1^* \end{pmatrix}. \tag{7}$$

Comparing (7) with (2), we find the unitary generator matrix

$$\underline{\mathbf{G}} = \begin{pmatrix} \underline{\mathbf{G}}[1] \\ \underline{\mathbf{G}}[2] \end{pmatrix} = \left(\begin{array}{cccc} 1 & j & 0 & 0 \\ 0 & 0 & 1 & j \\ \hline 0 & 0 & -1 & j \\ 1 & -j & 0 & 0 \end{array}\right). \tag{8}$$

The implicitly defined submatrices $\underline{\mathbf{G}}[k]$ characterize the k-th symbol of one codeword. It can easily be verified that the resulting system matrix $\mathbf{A}$ has orthogonal columns. Thus, maximum-likelihood decoding corresponds to simple matched filtering followed by a symbol-wise decision.

For the Alamouti code, $K_{ST}/N_{ST} = 1$ symbol is transmitted per channel use. However, for more than two transmit antennas there exists no linear orthogonal space-time block code with rate one. Some examples for high rate codes can be found in [7]. The transmission of one symbol per channel use for $N_T > 2$ is only possible if either the linearity or the orthogonality constraint is dropped. In [8], the following full rate full diversity quasi-orthogonal space-time code

$$\underline{\mathbf{X}} = \begin{pmatrix} \underline{d}_1 & -\underline{d}_2^* & -\tilde{\underline{d}}_3^* & \tilde{\underline{d}}_4 \\ \underline{d}_2 & \underline{d}_1^* & -\tilde{\underline{d}}_4^* & -\tilde{\underline{d}}_3 \\ \tilde{\underline{d}}_3 & -\tilde{\underline{d}}_4^* & \underline{d}_1^* & -\underline{d}_2 \\ \tilde{\underline{d}}_4 & \tilde{\underline{d}}_3^* & \underline{d}_2^* & \underline{d}_1 \end{pmatrix} = \begin{pmatrix} \underline{\mathbf{X}}_1 & -\underline{\mathbf{X}}_2^* \\ \underline{\mathbf{X}}_2 & \underline{\mathbf{X}}_1^* \end{pmatrix} \tag{9}$$

was proposed, where $\tilde{\underline{d}}_i = e^{j\theta}\underline{d}_i$ are rotated versions of the actual information symbols. For QPSK modulation, the optimum rotation angle was shown to be $\theta = \pi/6$. The corresponding 16×8 dimensional generator matrix $\underline{\mathbf{G}}$ can easily be derived from this similar to (8) and will not be stated explicitly here. Comparing (9) with (7) we see that the quasi-orthogonal code follows from a recursive application of the Alamouti scheme. Due to this structure, $\underline{d}_1$ only interferes with $\underline{d}_4$ and $\underline{d}_2$ with $\underline{d}_3$, respectively. Therefore, a maximum-likelihood detector can estimate these two symbol pairs independently, so the complexity is smaller than for general non-orthogonal schemes.

C. Multistratum Space-Time Codes

Multistratum space-time codes combine the benefits of both space-time block codes and V-BLAST, i.e. they exploit transmit diversity and at the same time allow for high data rates [3]. They are a special case of the more general class of linear dispersion codes [9]. As in V-BLAST, the information symbols are demultiplexed into $N_S \leq \min\{N_T, N_R\}$ parallel data streams, which are referred to as strata. Instead of directly transmitting these symbols, all strata are first encoded by the same space-time block code with generator matrix $\underline{\mathbf{G}}'$. Hence, the strata experiences full transmit diversity. Before superimposing the individual strata an orthogonal transform is applied, so that it is possible to separate them at the receiver.

An alternative interpretation of the encoding process is that each stratum employs its own characteristic space-time block code. The generator matrix for the k-th codeword symbol of the l-th stratum is given by

$$\underline{\mathbf{G}}_l[k] = w_{k-1,l-1}\,\underline{\mathbf{G}}'[k], \tag{10}$$

where $w_{k,l}$ is the kernel of the employed orthogonal transform. The DFT kernel is known to be $w_{k,l} = e^{-j2\pi kl/N_{ST}}$, while for the Hadamard transform (which only exists for $N_{ST} = 2^n$)

$$w_{k,l} = (-1)^{\sum_{i=0}^{n-1} b_i(k) b_i(l)} \tag{11}$$

must be used, where $b_i(x)$ denotes the i-th bit in the binary representation of x. With these definitions, the complete generator matrix of a multistratum space-time code becomes

$$\underline{\mathbf{G}} = \begin{pmatrix} \underline{\mathbf{G}}_1[1] & \dots & \underline{\mathbf{G}}_{N_S}[1] \\ \vdots & \ddots & \vdots \\ \underline{\mathbf{G}}_1[N_{ST}] & \dots & \underline{\mathbf{G}}_{N_S}[N_{ST}] \end{pmatrix} . \tag{12}$$

Taking the Alamouti scheme from (8) as an example, we get with the maximum number of $N_S = 2$ strata

$$\underline{\mathbf{G}} = \left(\begin{array}{cccc|cccc} 1 & j & 0 & 0 & 1 & j & 0 & 0 \\ 0 & 0 & 1 & j & 0 & 0 & 1 & j \\ \hline 0 & 0 & -1 & j & 0 & 0 & 1 & -j \\ 1 & -j & 0 & 0 & -1 & j & 0 & 0 \end{array} \right) . \tag{13}$$

In this paper we will focus on the quasi-orthogonal space-time code from (9), which allows for up to $N_S = 4$ strata. The corresponding overall generator matrix has dimension 16×32. Note that when using the DFT, the resulting system matrix $\mathbf{A}$ is rank deficient. Therefore, we take the Hadamard kernel (11), where this problem does not occur. There may be even better transforms, but the search for them goes beyond the scope of this paper.

In addition to the exploitation of transmit diversity, multistratum codes offer another important advantage compared to V-BLAST, as they can easily adapt to situations where $N_R < N_T$ (or the receive antennas are highly correlated) by switching off some of the strata without transmission breaks. This enables a flexible trade-off between data rate and error performance. The special case $N_S = 1$ corresponds to ordinary space-time block coding.

V. Detection Algorithm

Assuming perfect channel knowledge at the receiver, ML detection corresponds to the minimization of the squared Euclidean distance $\|\mathbf{y} - \mathbf{A}\mathbf{d}'\|^2$ over all $M^{K_{ST}}$ possible vectors $\mathbf{d}'$. Introducing the QL decomposition of the system matrix $\mathbf{A} = \mathbf{QL}$, where $\mathbf{Q}$ has orthonormal columns and $\mathbf{L}$ is lower triangular, it is obvious that minimizing

$$\|\mathbf{z} - \mathbf{L}\,\mathbf{d}'\|^2 = \sum_{i=1}^{N} \left| z_i - \sum_{k=1}^{i} l_{i,k}\, d'_k \right|^2 \tag{14}$$

with $\mathbf{z} = \mathbf{Q}^T\mathbf{y}$ and $N = 2K_{ST}$ is equivalent to the original ML problem. In contrast to the brute force approach, the sphere decoder restricts the search space to a ball of radius R around the projected receive vector $\mathbf{z}$ [10]. Note that the sum in (14) can be calculated recursively by

$$\Delta_i = \Delta_{i-1} + \left| z_i - \sum_{k=1}^{i} l_{i,k}\, d'_k \right|^2 \quad \text{with} \quad \Delta_0 = 0\,. \tag{15}$$

Since the increment in (15) is always nonnegative, we have $\Delta_i \le \Delta_N$. Thus, a point $\mathbf{d}'$ can only be valid if the condition $\Delta_i < R^2$ is fulfilled for all $1 \le i \le N$. Consequently, in a first step d'_1 is chosen such that $\Delta_1 = |z_1 - l_{1,1}d'_1|^2 < R^2$ holds. Keeping this value fixed, the algorithm proceeds analogously with d'_2 up to d'_N. If for some index i the partial sum exceeds the squared radius another hypothesis will be tested for d'_{i-1}, and so on. This procedure corresponds to a depth-first search of the decision tree.

0) Initialization
   $i = 0\,, \quad \Delta_0 = 0\,, \quad R^2 = \infty$

1) Successive Interference Cancellation
   while $i < N$ and $\Delta_i < R^2$
      $i = i + 1$
      $\tilde{d}_i = \frac{1}{l_{i,i}} \left(z_i - \sum_{k=1}^{i-1} l_{i,k}\, d'_k \right)$
      $d'_i = \mathcal{Q}\{\tilde{d}_i\}$
      $\Delta_i = \Delta_{i-1} + |l_{i,i}(\tilde{d}_i - d'_i)|^2$
   end

2) Check if new best point found
   if $\Delta_N < R^2$
      $\hat{\mathbf{d}} = \mathbf{d}'$
      $R^2 = \Delta_N$
   end

3) Look for other possible points
   while $i > 1$
      $i = i - 1$
      try next best estimate for d'_i (cf. Fig. 2)
      $\Delta_i = \Delta_{i-1} + |l_{i,i}(\tilde{d}_i - d'_i)|^2$
      if $\Delta_i < R^2$
         goto step 1)
      end
   end

Fig. 1. Pseudo-code of the basic sphere decoding algorithm

The original version by Fincke and Pohst picks the values for d'_i in ascending order from the set of feasible symbols [10]. It is mandatory to initialize the search radius appropriately. One possibility is to start with a small radius and rerun the whole algorithm with increased R if no point lies inside the sphere [11]. A complexity analysis for system matrices with uncorrelated Gaussian entries can be found in [12].

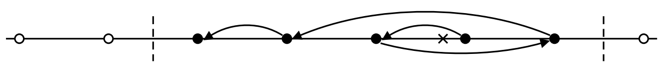

Fig. 2. Illustration of the Schnorr-Euchner enumeration strategy. The cross represents $\tilde{d}_i$, and the black dots are feasible values for d'_i such that $\Delta_i < R$.

The pseudo-code given in Fig. 1 is based on the improved Schnorr-Euchner enumeration strategy [13], which was also applied in [4]. As illustrated in Fig. 2, symbols close to the decision variable $\tilde{d}_i$ are considered first. This increases the probability of early finding the ML estimate. In order to avoid restarts, we set $R^2 = \infty$ in the initialization step. Hence, the first vector $\hat{\mathbf{d}}$ is the result of an ordinary SIC, which is also referred to as Babai point. The radius is adjusted every time a new point is found.

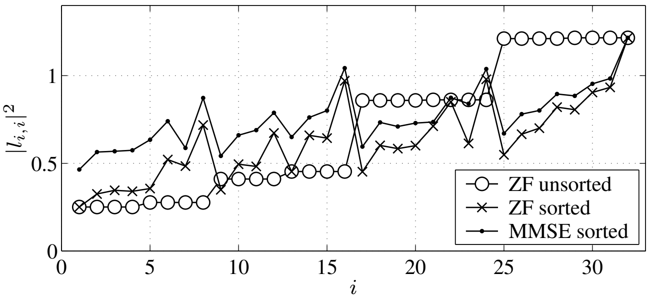

Fig. 3. Influence of sorting and MMSE filtering on $|l_{i,i}|^2$

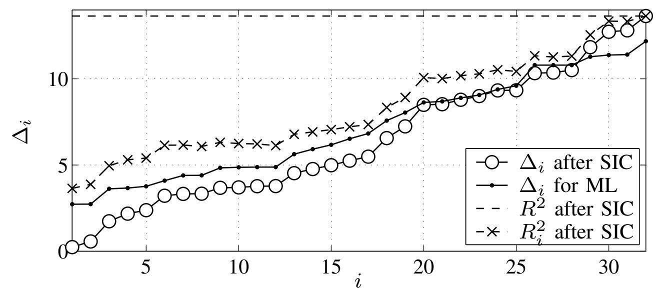

Fig. 4. Cumulative Distances Δ_i after initial SIC and for ML solution. Also shown are the initial squared radii R^2 and R_i^2 for $\alpha_1 = 8$ and $\alpha_N = 1$.

A. Optimizing the Detection Order

It is well known that the order of detection is crucial for the performance of SIC. Although SD always leads to the optimum ML estimate, the effort highly depends on the result of the initial interference cancellation step. Furthermore, if an error occurs in an early stage many paths of the decision tree will be searched in vain. Thus, it is strongly recommended to also optimize the detection sequence when using SD. This has already been suggested in [14], where the greedy sorting algorithm from [15] was used. An even more efficient method was proposed in [5]. As sorting has to be performed only once per frame, the computational overhead is negligible.

In [16], a complex version of the SD algorithm was introduced that is supposed to have lower complexity. However, this is not the case if the structure of the equivalent real system matrix, i.e. the orthogonality between real and imaginary part of one symbol, is properly exploited. Moreover, the real-valued representation offers more degrees of freedom for the optimization of the detection order [17].

B. MMSE Interference Suppression

From (14) it follows that the signal z_i in the filter output vector $\mathbf{z} = \mathbf{Q}^T\mathbf{y}$ contains no interference from $d_{i+1}, \ldots, d_N$. This zero-forcing (ZF) solution may cause strong noise amplification, especially for ill-conditioned system matrices $\mathbf{A}$. The performance of SIC and hence the speed of SD can be significantly improved by using MMSE interference suppression instead [5], [14]. It was shown in [15] that the desired MMSE filter corresponds to the matrix $\mathbf{Q}_1$ (with the same dimension as $\mathbf{A}$) in the QL decomposition of the extended system matrix

$$\begin{pmatrix} \mathbf{A} \\ \frac{\sigma_n}{\sigma_d}\mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \mathbf{L} \,. \tag{16}$$

Hence, the SD algorithm must be run with $\mathbf{z} = \mathbf{Q}_1^T\mathbf{y}$. Note that this is not identical to ML detection anymore, because in $\mathbf{z}$ there remains interference of subsequent symbols and the noise is colored.

Fig. 3 shows an example for the squared magnitudes of the diagonal elements $l_{i,i}$, which are proportional to the signal to interference plus noise ratio (SINR) in the corresponding detection steps. Without sorting, the first eight decisions are very unreliable. This is because the first stratum experiences strong interference from the remaining ones. Sorting increases $|l_{i,i}|^2$ in this range, but inevitably leads to smaller SINR in the late stages. MMSE filtering results in further remarkable improvements.

C. Adaption of Radii

From Fig. 4 it can be observed that the inequality $\Delta_i \leq R^2$ is very loose for small i. Therefore, it would be advantageous to replace R by an appropriate sequence of radii R_i. This corresponds to pruning the decision tree, i.e. some possible but unlikely branches are disregarded. While ML performance can not be guaranteed anymore, the complexity may be lowered considerably.

A statistical pruning strategy was proposed in [18]. For the calculation of R_i, the system matrix was assumed to be uncorrelated, which is not the case for multistratum space-time codes. Moreover, as the radii are predetermined, restarts may be required if no feasible solution is found. Therefore, we here suggest a heuristic approach that does not suffer from these drawbacks. The main idea is to adapt R_i in step 2) of the algorithm in Fig. 1 according to the cumulative distances Δ_i, so we set

$$R_i^2 = \min\{\Delta_i + \delta_i, \Delta_N\} \,. \tag{17}$$

The gaps δ_i depend on the average distance increment in each step and are chosen to decrease linearly from $\delta_1 = \alpha_1 \frac{\Delta_N}{N}$ to $\delta_N = \alpha_N \frac{\Delta_N}{N}$. This is motivated by the fact that Δ_i increases for small i while it decreases for large i during the iterations of the SD algorithm (cf. Fig. 4). The parameters α_1 and α_N allow a tradeoff between error performance and computational effort. For large values the pruning is inefficient, whereas the ML estimate may not be obtained if they are too small.

VI. Simulation Results

In this section we present some simulation results in order to demonstrate the efficiency of the proposed schemes. We employed QPSK modulation and the full rate quasi-orthogonal multistratum space-time code for $N_T = 4$ transmit antennas from Section IV-C. Hence, the data rate is 8 bit per channel use. The receiver is also equipped with four antennas.

In Fig. 5, the multistratum code is compared to V-BLAST. Surprisingly, after simple SIC with sorting and MMSE filtering, V-BLAST is much better. This is due to the very strong interference between strata and the resulting low SINR

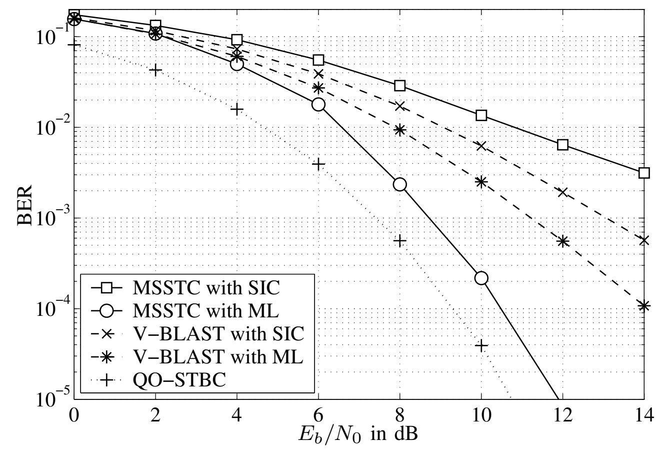

Fig. 5. Bit error rates for MSSTC and V-BLAST with SIC and ML detection. The quasi-orthogonal space-time block code from (9) serves as reference.

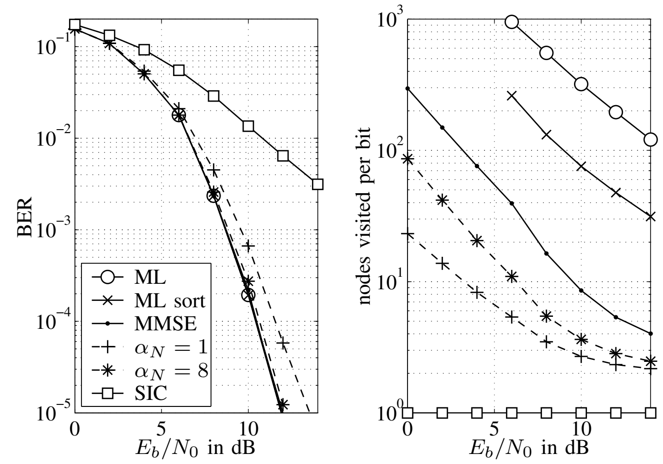

Fig. 6. Bit error rates and complexity of detection algorithms ($\alpha_1 = 8$).

in the first detection steps already observed in Section V-B. However, with ML decoding the MSSTC clearly outperforms V-BLAST because of the additional transmit diversity. This motivates the need for efficient ML detection algorithms in this case. A comparison with the quasi-orthogonal space-time code from (9) reveals that the multistratum code also achieves full diversity.

Fig. 6 shows bit error rates and complexities for the proposed modifications of the original sphere decoder. As measure for the complexity we use the number of nodes of the decision tree visited during the detection process. While sorting already significantly reduces the computational effort, further extremely large reductions are achieved by MMSE filtering without sacrificing performance. Moreover, properly adapting the radii R_i allows for a very good compromise between achievable bit error rates and the corresponding complexity. With $\alpha_1 = \alpha_N = 8$ the performance degradation is marginal, while $\alpha_N = 1$ may be chosen for low signal to noise ratio (SNR). The resulting complexity is comparable to that of SIC.

VII. Conclusion

In this paper, a quasi-orthogonal multistratum space-time code achieving full rate with full diversity was introduced. We demonstrated that SIC is far from being optimum, so efficient (near) ML decoding is required. Besides using an optimized detection order and MMSE interference suppression in the SD algorithm, which was already suggested in previous publications, we also proposed a tree pruning strategy that can further reduce the computational effort, especially for low SNR. Our new SD variant is not limited to multistratum space-time codes. Additional improvements may be possible if the structure of the system matrix $\mathbf{A}$ is taken into account.

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