Blind Identification and Equalization of LDPC-encoded MIMO Systems

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***Abstract*— In this paper we propose a new algorithm which blindly identifies and equalizes a MIMO system, where all sources are independently protected against errors by an LDPC-Code. To this end the proposed method exploits statistical dependencies caused by the channel code. In contrast to most common blind source separation algorithms, the new method does not suffer from a permutation ambiguity. Furthermore, if the channel code is asymmetric, the suggested method delivers phase correct estimates of the channel and the corresponding equalizer. The performance of the presented method will be evaluated by numerical results.**

I. Introduction

Low density parity check (LDPC) codes are known for their excellent error correction capabilities in context with moderate decoding complexity. As demonstrated in several contributions, e.g. [1], [2], near Shannon limit performance can be achieved by simple decoding techniques as e.g. the sum-product algorithm [3]. Due to the low number of bits, which have to be taken into account in each parity check equation, LDPC codes are also well suited for blind deconvolution and channel estimation techniques [4]. This paper addresses the issue of blind system identification and equalization of MIMO systems. The proposed method exploits statistical dependencies caused by a LDPC code in order to blindly identify the channel and simultaneously adjust a linear equalizer.

Most blind source separation (BSS) methods as e.g. [5], [6] are based on the assumption of statistically independent and non gaussian distributed sources. The channel causes a linear superposition of these sources. Thus, as stated by the central limit theorem, the distribution of the channel output is more ”gaussian” than the channel input. The key idea behind blind source separation is to design blindly (without channel state information) a linear equalizer so that the equalizer restores the statistical independence by decorrelating the equalizer's output and making it as less ”gaussian” as possible.

This class of BSS methods suffers from a phase and permutation ambiguity with respect to the estimated channel impulse response and the corresponding linear equalizer, since the gaussian target criterion is invariant of a complex factor and the equalizer's outputs can not be uniquely assigned to the sources. In order to resolve the phase ambiguity, several contributions [7], [8] have utilized asymmetric mapping constellations. We will show that even with symmetric mapping constellation the phase can be uniquely determined, if the channel code satisfies an asymmetric condition. Furthermore, the channel code can be utilized to make each layer distinguishable.

This paper is organized as follows: In Section II the system model will be presented. Since channel coding plays a key role for the proposed method, we will discus some required properties of channel codes and give some definitions in Section III. The presented algorithm will be derived in Section IV. Finally we give numerical results in Section V and conclude this paper in Section VI.

II. System Model

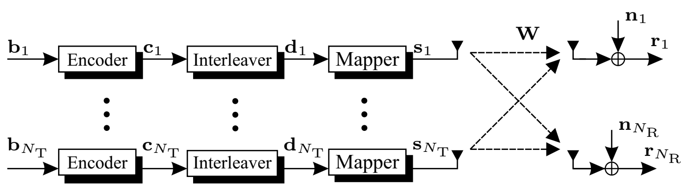

Fig. 1. System Model

As illustrated in Fig. 1, we consider a layered block transmission of N_T independent binary data streams $\mathbf{b}_m \in \{0,1\}^{(I\times 1)}$ of length I, where $m = 1, \cdots, N_\mathrm{T}$ is the layer index. After encoding each layer independently by a linear block encoder, we obtain

$$\mathbf{c}_m = \mathrm{mod}(\mathbf{G}\mathbf{b}_m, 2), \qquad (1)$$

where $\mathbf{G} \in \{0,1\}^{(I\times K)}$ is the channel code generator matrix with rate $R_c = I/K < 1$ and $\mathbf{c}_m \in \{0,1\}^{K\times 1}$. Afterwards, at each layer the resulting bits are permuted by

$$\mathbf{d}_m = \mathbf{P}_m\mathbf{c}_m, \qquad (2)$$

where the permutation matrix $\mathbf{P}_m \in \{0,1\}^{K\times K}$ contains exactly one nonzero element in each row and column. Encoding and permutation can be summarized by the overall generator matrix $\tilde{\mathbf{G}}_m = \mathbf{P}_m\mathbf{G}$. Please note, that the generator matrix of a certain layer may coincide with any other, but the permutation matrix of each layer should differ, in order to make each layer distinguishable from any other.

After mapping the bits into the signal space by BPSK modulation assigning $0 \rightarrow 1$ and $1 \rightarrow -1$, the channel input is given by

$$\mathbf{s}_m = \mathrm{Map}_{\mathrm{BPSK}}(\mathbf{d}_m). \quad (3)$$

This data can be arranged in the matrix $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_{N_\mathrm{T}}]^T = [\mathbf{s}(1), \cdots, \mathbf{s}(K)]$. The signal pass through a $(N_\mathrm{R} \times N_\mathrm{T})$ MIMO channel with $N_\mathrm{R} \geq N_\mathrm{T}$. Collecting N_R samples at instance k the channel output can be expressed as

$$\mathbf{r}(k) = \mathbf{W}\mathbf{s}(k) + \mathbf{n}(k) \quad (4)$$

where $\mathbf{W} = [\mathbf{w}_1, \cdots, \mathbf{w}_{N_\mathrm{T}}]$ is the MIMO channel impulse response (CIR) and $\mathbf{n}(k) \in \mathbb{C}^{N_\mathrm{R}\times 1}$ is i.i.d. white gaussian noise with covariance $\mathrm{E}\{\mathbf{n}(k)\mathbf{n}^H(k)\} = \sigma_n^2 \mathbf{I}_{N_\mathrm{R}}$.

III. Channel Coding

Since they are well suited for our purpose, throughout this paper we consider regular low density parity check (LDPC) codes for channel coding. A LDPC code is characterized by a sparse parity check matrix $\mathbf{H}$ of dimension $(L \times K)$, where the number of nonzero elements Ω in $\mathbf{H}$ is much lower than the number of zeros. The number of rows in $\mathbf{H}$ is at least $L \geq K - I$ and $\mathbf{H}$ has rank $K - I$. The relation

$$\mathrm{mod}(\mathbf{HG}, 2) = \mathbf{0} \quad (5)$$

holds with respect to the corresponding generator matrix. Due to (5) the parity check sum equation

$$\mathrm{mod}(\mathbf{Hc}, 2) = \mathbf{0} \quad (6)$$

also holds for any encoded data bit stream $\mathbf{c} = \mathrm{mod}(\mathbf{Gb}, 2)$.

A LDPC code is called regular, if the number of 1's in each row as well as the number of 1's in each column is exactly equal. Let ρ denote the number of 1's in each row called right degree and λ the number of 1's in each column called left degree. Please note that the set $\mathcal{A}$ of all valid code words is uniquely defined by $\mathbf{H}$, whereas several generator matrices exist fulfilling (5).

Let $\mathcal{H}_l = \{\kappa_{l,1}, \cdots, \kappa_{l,\rho}\}$ be the set of column indices according to the nonzero elements in the l-th row of $\mathbf{H}$. Then, analogue to (6) for $l = 1, \cdots, L$ the parity check equation

$$c(\kappa_{l,1}) \oplus \cdots \oplus c(\kappa_{l,\rho}) = 0 \quad (7)$$

is satisfied, where $\oplus$ is the XOR-operator. After mapping the bits into the signal space by BPSK (bit assignment as in section II), the XOR-operator can be replaced by the multiplication, i.e. the signal space representation of (7) is given by

$$\prod_{k \in \mathcal{H}_l} s(k) = 1. \quad (8)$$

Please note that for any nonmember of $\mathcal{A}$ the expectation of the product becomes

$$\mathrm{E}\Big\{ \prod_{k \in \bar{\mathcal{H}}} s(k) \Big\} = 0, \quad (9)$$

where $\bar{\mathcal{H}}$ is an arbitrary set of indices according to the null space of the code. Since we aim to obtain phase correct channel estimates, the signal space representation of any valid codeword should be unique independently from a complex factor. To this end the channel code should be asymmetric as stated in the following definition:

Definition 1: A channel code is called *asymmetric*, if the negation of an arbitrary valid code word is not a valid code word:

$$\mathbf{c} \in \mathcal{A} \Leftrightarrow \overline{\mathbf{c}} \notin \mathcal{A}$$

Obviously, a code is asymmetric if an arbitrary parity check sum includes an odd number of encoded bits, e.g.

$$c(1) \oplus c(2) \oplus c(3) = 0 \Leftrightarrow \overline{c(1)} \oplus \overline{c(2)} \oplus \overline{c(3)} \neq 0, \quad (10)$$

where the overbar denotes the negation of the bit. Thus, we can state following theorem:

Theorem 1: Let $\mathbf{H}$ be the parity check matrix of a linear channel code. If there exists a row or a linear combination of rows in $\mathbf{H}$ such that the number of 1's is odd, then this code is asymmetric.

We call a code to be *strong* asymmetric, if each row of $\mathbf{H}$ includes an odd number of nonzero elements. Consequently, we restrict in the following to regular LDPC codes with odd ρ.

In the next section we make use of the following set definition: Let $l(\tilde{l}, k)$ be the row of $\tilde{l}$-th nonzero element in the k-th column in $\mathbf{H}$. After excluding k from $\mathcal{H}_{\tilde{l}}$, the number of the remaining elements is even. Thus, the remaining part can be separated into 2 equal sized subsets $\mathcal{H}_{\tilde{l},k}^{(1|2)}$ and $\mathcal{H}_{\tilde{l},k}^{(2|2)}$.

IV. Algorithm

This section is separated into two subsections. The focus of first subsection is on the derivation of the proposed algorithm for blind equalization and channel estimation with respect to a single layer, whereas the second subsection deals with issue of multilayer detection.

A. Blind Equalization and Channel Estimation

Since in this section we focus on estimating the equalizer and the channel only with respect to the m-th layer, for the sake of clarity the layer index m is dropped in some notations.

Let $\mathbf{e} \in \mathbb{C}^{N_\mathrm{R}\times 1}$ be an arbitrary linear filter unequal zero,

$$y(k) = \mathbf{e}^H \mathbf{r}(k) \quad (11)$$

be the filter output at the receiver and $\mathbf{q} = \mathbf{W}^H\mathbf{e}$ be the overall impulse response of filter and channel. Furthermore, assume that $\mathbf{H}$ is the parity check matrix corresponding to the generator matrix $\tilde{\mathbf{G}}_m$ of the m-th layer. Selecting an arbitrary 1 in the parity check matrix $\mathbf{H}$, e.g. the $\tilde{l}$-th 1 in the k-th column, it can be shown that the m-th column $\mathbf{w}_m$ of the channel impulse response $\mathbf{W}$ weighted by some real positive factor is given by the expectation

$$\mathrm{E}\Big\{\mathbf{r}(k) \prod_{\mu\in\mathcal{H}^{(1|2)}_{\tilde{l},k}} y(\mu) \prod_{\nu\in\mathcal{H}^{(2|2)}_{\tilde{l},k}} y^*(\nu)\Big\} \tag{12}$$

$$= \mathrm{E}\Big\{\big(\mathbf{w}_m s_m(k) + \sum_{\tilde{m}\in\{1,\cdots,N_\mathrm{T}\}/m} \mathbf{w}_{\tilde{m}} \mathbf{s}_{\tilde{m}}(k) + \mathbf{n}(k)\big) \prod_{\mu\in\mathcal{H}^{(1|2)}_{\tilde{l},k}} \big(q_m^* s_m(\mu) + \sum_{\tilde{m}\in\{1,\cdots,N_\mathrm{T}\}/m} q^*_{\tilde{m}} s_{\tilde{m}}(\mu) + \eta(\mu)\big) \prod_{\nu\in\mathcal{H}^{(2|2)}_{\tilde{l},k}} \big(q_m s_m^*(\nu) + \sum_{\tilde{m}\in\{1,\cdots,N_\mathrm{T}\}/m} q_{\tilde{m}} s^*_{\tilde{m}} + \eta^*(\nu)\big)\Big\},$$

where $\eta(k) = \mathbf{e}^H \mathbf{n}(k)$ and q_m is the m-th entry of $\mathbf{q}$.

Proof: Interchanging the order of products and sums (12) can be rearranged to (13) in the bottom line of the next page, where $\mathcal{M}$ is the set of ρ-tuples with cardinality N_T^ρ consisting of all layer combinations of length ρ and $\mathcal{H}_{\tilde{l}(l,k)} = \{\kappa_{\tilde{l},1}, \cdots, \kappa_{\tilde{l},\rho}\}$. Since the noise is i.i.d, all elements incorporating noise vanish in (13). Assuming that no parity check sum is accidentally caught in $v(\tilde{m}_1, \cdots, \tilde{m}_\rho)$, due to (9) this term becomes zero and only the left term remains so that

$$\mathrm{E}\Big\{\mathbf{r}(k) \prod_{\mu\in\mathcal{H}^{(1|2)}_{\tilde{l},k}} y(\mu) \prod_{\nu\in\mathcal{H}^{(2|2)}_{\tilde{l},k}} y^*(\nu)\Big\} = \mathbf{w}_m |q_m|^{\rho-1}. \tag{14}$$

Obviously, (14) is equivalent to the true channel coefficients $\mathbf{w}_m$ corresponding to the m-th layer, which is weighted by a real positive factor $|q_m|^{\rho-1}$. This factor is real and positive, since the phase rotation caused by the product according to the subset $\mathcal{H}^{(1|2)}_{\tilde{l},k}$ is compensated by the conjugated counterpart according to $\mathcal{H}^{(2|2)}_{\tilde{l},k}$. Recall that (14) holds for Ω different sets $\mathcal{H}^{(1|2)}_{\tilde{l},k}$ and $\mathcal{H}^{(2|2)}_{\tilde{l},k}$ according to the number of 1's in $\mathbf{H}$.

In real applications the expectation in (12) has to be approximated. This can be done by averaging over Ω different equations of this type, i.e.

$$\hat{\mathbf{w}}_m = \frac{1}{\Omega} \sum_{\tilde{l}=1}^{\lambda} \sum_{k=1}^{K} \mathbf{r}(k) \prod_{\mu\in\mathcal{H}^{(1|2)}_{\tilde{l},k}} y(\mu) \prod_{\nu\in\mathcal{H}^{(2|2)}_{\tilde{l},k}} y^*(\nu) \tag{15}$$

$$= \mathbf{w}_m |q_m|^{\rho-1} + \sum_{\substack{(\tilde{m}_1\cdots\tilde{m}_\rho) \\ \in\mathcal{M}\setminus(m\cdots m)}} u(\tilde{m}_1, \cdots, \tilde{m}_\rho) \hat{v}(\tilde{m}_1, \cdots, \tilde{m}_\rho) + \tilde{\eta}(k),$$

where $\tilde{\eta}(k)$ summarizes all parts in $\hat{\mathbf{w}}_m$ incorporating noise and

$$\hat{v}(\tilde{m}_1, \cdots, \tilde{m}_\rho) = \frac{1}{\Omega} \sum_{\tilde{l}=1}^{\lambda} \sum_{k=1}^{K} \prod_{\gamma=1}^{\rho} s_{\tilde{m}_\gamma}(\kappa_{\tilde{l},\gamma}). \tag{16}$$

Unfortunately, in contrast to $v(\tilde{m}_1, \cdots, \tilde{m}_\rho)$ this term may not become exactly zero. The remaining error is weighted by $u(\tilde{m}_1, \cdots, \tilde{m}_\rho)$, where the power of this factor depends on the current filter adjustment. If $\mathbf{e}$ is an ideal linear equalizer with respect to the m-th layer so that $\mathbf{q} = \mathbf{W}\mathbf{e}$ consists of exactly a single one at position m, the term $u(\tilde{m}_1, \cdots, \tilde{m}_\rho)$ becomes zero for $(\tilde{m}_1 \cdots \tilde{m}_\rho) \in \mathcal{M} \setminus (m \cdots m)$. Hence, a readjustment of the filter $\mathbf{e}$ on the basis of the current channel estimate may assist reducing the impact of this error. Therefore, we suggest an iterative two-step algorithm, where channel estimation and filter adaptation are repeated alternately until the algorithm converges. Let i be an iteration counter, $\mathbf{e}_{(0)} = [1, \cdots, 1]^T$ be the initial equalizer at iteration $i = 0$, and $\mathbf{w}_m^{(i)}$ an channel estimate of the i-th step according to (15). On the basis of the channel estimate the filter's coefficients can be adjusted, e.g. by the MMSE approach with

$$\tilde{\mathbf{e}}_{(i+1)} = \mathbf{\Phi}_{rr}^{-1} \hat{\mathbf{w}}_m^{(i)}, \tag{17}$$

where $\mathbf{\Phi}_{rr} = 1/K \sum_{k=1}^{K} \mathbf{r}(k)\mathbf{r}^H(k)$ is an estimate of the covariance matrix of the receive signal. In order to avoid a bit overflow, $\tilde{\mathbf{e}}_{(i)}$ should be normalized by

$$\mathbf{e}_{(i)} = \frac{\tilde{\mathbf{e}}_{(i)}}{\sqrt{\tilde{\mathbf{e}}_{(i)}^H \mathbf{\Phi}_{rr} \tilde{\mathbf{e}}_{(i)}}}. \tag{18}$$

The algorithm is summarized in Tab. I.

TABLE I

PHASE CORRECT BLIND DECONVOLUTION EXPLOITING CHANNEL CODING (BDCC)

1	:	Initialize $\mathbf{e}_{(0)} = [1, \cdots, 1]^T$ and the iteration counter $i = 0$.
2	:	**repeat**
3	:	Estimate the channel by (15).
4	:	Update the linear equalizer by (17) and (18).
5	:	Set $i = i + 1$.
6	:	**end**

B. Successive Interference Cancellation

In order to deal with the complete number of layers we propose a successive interference cancellation scheme as illustrated in Fig. 2. At the first stage the layer index is initialized to $m = 1$. As described in the previous section the blind estimator delivers equalizer coefficients as well as an channel estimate with respect to the m-th layer. After equalizing, demapping, deinterleaving and decoding $\mathbf{r}(k)$, an estimate of the information bearing sequence $\hat{b}_m(i)$ is obtained at the receiver end. In order to mitigate the influence of the m-th layer, the signal $\hat{s}_m(k)$ has to be reconstructed by encoding, interleaving and mapping $\hat{b}_m(i)$ again into the signal space. The product $\hat{s}_m(k)\hat{\mathbf{w}}_m$ is subtracted from $\mathbf{r}(k)$ and the layer index m is incremented. These procedure will be repeated until all layers are processed.

V. NUMERICAL RESULTS

In our simulations the sources were encoded by a random generated regular (ρ, λ)-LDPC code without 4-circles. The encoded signal were transmitted over an (4×4) block fading channel with i.i.d. zero mean complex gaussian distributed channel gains. Fig. 3 and Fig. 4 shows the NMSE performance

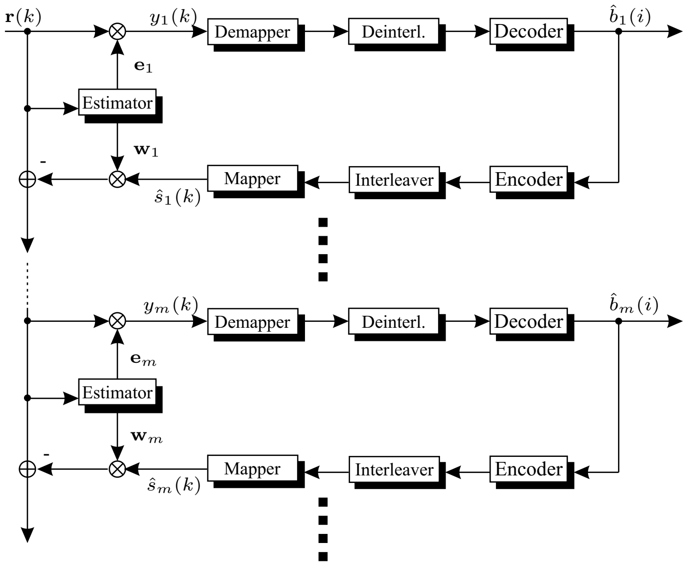

Fig. 2. Successive interference cancellation

versus the E_b/N_0 ratio of the blind deconvolution exploiting channel coding (BDCC), where the normalized mean squared error (NMSE) between the true and the estimated channel is defined by

$$\mathrm{NMSE} = \frac{\|\mathbf{w}_m - \hat{\mathbf{w}}_m\|^2}{\|\mathbf{w}_m\|^2}. \tag{19}$$

In Fig. 3 the impact of the block length K on the NMSE-performance is examined for $(2,3)$-LDPC codes. The NMSE-performance benefits from large blocklength. All curves in 3 shows an error floor at high E_b/N_0. The larger the blocklength the more is the starting point of the error floor shifted to high E_b/N_0-value. However, the NMSE values are even considerable for block length $K = 100$.

In Fig. 4 it can be observed, that the performance of the BDCC depends strongly on the right degree ρ of the particular channel coding. The reason may be on the one hand that the number of terms, which are accumulated in (13), becomes very high for large ρ and consequently the performance is very sensitive. On the other hand also the impact of noise on the estimation must be taken into account. For small ρ the BDCC performs very well.

In Fig. 5 the BER for each layer versus E_b/N_0 is plotted, where a $(2,3)$-LDPC code with a blocklength 100 were used. In order to remove fading effects, in this simulation power of each column in the channel matrix $\mathbf{W}$ were normalized. It can be observed that the latter layers benefit from the successive interference cancellation, whereas the former layer suffer from multilayer interference. Analogue to the observation in Fig. 3 also here an error floor at high E_b/N_0-value can be observed.

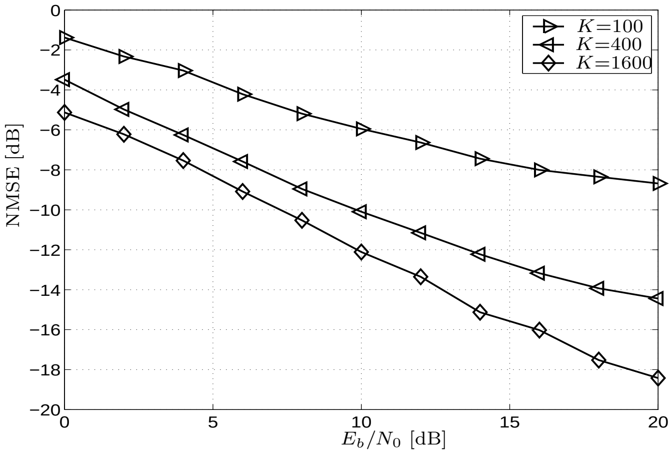

Fig. 3. NMSE vs. E_b/N_0 for $(2,3)$-LDPC code with block length $K = 100$, $K = 400$, $K = 1600$

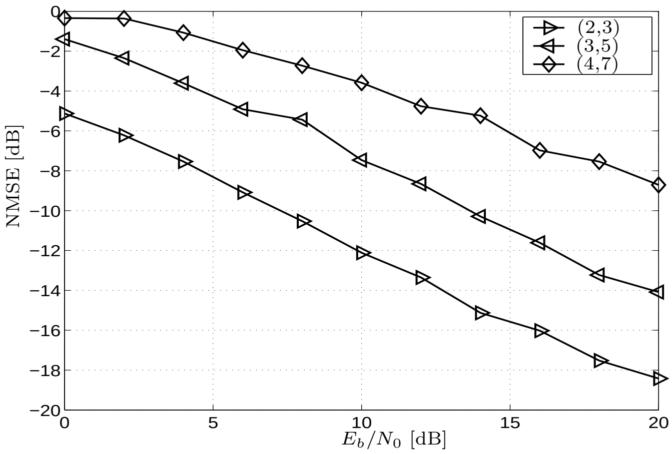

Fig. 4. NMSE vs. E_b/N_0 for $(2,3)$-, $(3,5)$- and $(4,7)$-LDPC codes with block length $K = 1600$

VI. Conclusions

In this paper we have derived an iterative algorithm, which blindly identifies a MIMO channel exploiting the statistical dependencies of the transmitted signal caused by channel coding. The algorithm jointly updates channel estimates and adapts a linear equalizer. In order to perform multilayer detection a successive interference scheme was proposed. It was shown that the detected layer can be uniquely assigned to the source by proper channel code design. For asymmetric channel

$$\mathbf{w}_m |q_m|^{\rho-1} \underbrace{\mathrm{E}\Big\{\prod_{k\in\mathcal{H}_l} s_m(k)\Big\}}_{=1} + \sum_{\substack{(\tilde{m}_1\cdots\tilde{m}_\rho)\\ \in\mathcal{M}\setminus(m\cdots m)}} \underbrace{\mathbf{w}_{\tilde{m}_1} \prod_{\alpha=2}^{\rho/2} q_{\tilde{m}_\alpha} \prod_{\beta=\rho/2+1}^{\rho} q^*_{\tilde{m}_\beta}}_{u(\tilde{m}_1,\cdots,\tilde{m}_\rho)} \underbrace{\mathrm{E}\Big\{\prod_{\gamma=1}^{\rho} s_{\tilde{m}_\gamma}(\kappa_{\tilde{l},\gamma})\Big\}}_{v(\tilde{m}_1,\cdots,\tilde{m}_\rho)} \tag{13}$$

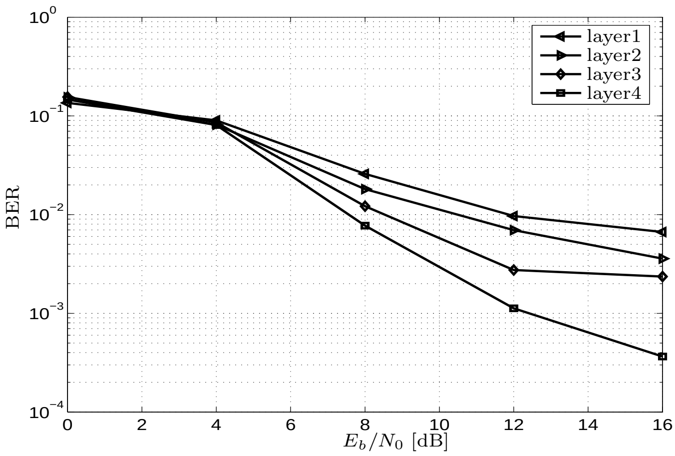

Fig. 5. BER vs. E_b/N_0 for $(2,3)$-LDPC code with block length $K = 100$

coding, the algorithm also delivers phase correct estimates. By simulations the performance of the proposed algorithm was examined. The NMSE-performance strongly depends on code properties, i.e. the right degree of its parity check matrix. In the case of a short right degree the algorithm performs quite good.

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