Pilot Aided Channel Estimation for Short-Code DS-CDMA

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Abstract

This paper investigates pilot based channel estimation techniques for short code DS-CDMA in a multipath environment. It will be shown that the conventional correlative estimation algorithm is biased and very sensitive to multi-user interference (MUI). Therefore, we suggest two advanced estimation algorithms. The first one eliminates the inherent path cross talk and yields an unbiased estimate while still suffering from MUI. The second approach suppresses the influence of the MUI by incorporating a linear interference suppression into the channel estimation algorithm. It will be shown that significant performance improvements can be achieved with the proposed algorithms especially under extreme near-far conditions.

DS-CDMA, Channel Estimation, Pilot Aided Channel Estimation

1 Introduction

CDMA is used in actual standards like UMTS [1] as a multiple access technique. In contrast to conventional multiple access schemes like TDMA and FDMA, two problems must be taken into account.

First, frequency selective channels destroy the orthogonality of code sequences which cause multi-user interference (MUI). There are a lot of techniques known suppressing this interference, i.e. linear filter structures like the decorrelator and the MMSE detector [2].

Second, the channel estimation in the presence of MUI is a severe problem. The easiest way to find an estimation of the channel impulse response (CIR) yields a structure similar to the RAKE receiver [3, 4]. This technique has the drawback of large variance caused by the MUI and it is biased for Short-Code CDMA.

An alternative solution is provided by blind techniques like [5],[6] and [7]. For a large number of active users in a CDMA system, this algorithm may exceed the conventional correlative channel estimation [8].

2 System Model

Throughout this paper, we assume a synchronous DS-CDMA-System. It can easily be extended to the asynchronous case by some modification to the system model [5]. A DS-CDMA-System can be expressed by

$$r(t) = \sum_{u=1}^{U} y_u(t) + n(t) \ , \tag{1}$$

where u denotes the user index and $n(t)$ is additive white gaussian zero mean noise with variance σ_n^2. The signal of user u can be written as

$$y_u(t) = \sum_{m} b_u(m) h_u(t - mT_s) * c_u(t) \ , \tag{2}$$

where "$*$" denotes the time continuous convolution, $b_u(m)$ is the m-th PSK modulated data symbol of the u-th user with variance $\sigma_b^2 = 1$, T_s the symbol duration, $c_u(t)$ the spreading sequence and $h_u(t)$ the CIR. The spreading sequence is defined by

$$c_u(t) = \sum_{k=1}^{K} c_{u,k} \dot{g}_{TF}(t - kTc) \tag{3}$$

with $g_{TF}(t)$ as transmission filter of length T_c with $T_c \ll T_s$ ($g_{TF}(t) = 0$ for $t < 0$ and $t > T_c$). K is the spreading factor with $KT_c = T_s$. The code coefficients $c_{u,k}$ should be elements of the set $[+1/\sqrt(K), -1/\sqrt(K)]$. It is convenient to summarize the convolution of CIR and spreading sequence to a signature $s_u(t)$ by

$$s_u(t) = c_u(t) * h_u(t) = \int_{\tau=\infty}^{\infty} c_u(\tau) h_u(t-\tau) d\tau \ . \tag{4}$$

In practical systems it can be assumed that the duration $T_{u,h}$ of the channel impulse response is shorter than the symbol period T_s. Let $L_{u,h}$ denote a discrete time representation of $T_{u,h}$ with $L_{u,h}T_c = T_{u,h}$. Introducing a sliding observation window of size $\tilde{K}T_c$, we get a discrete time matrix representation by comprising $\tilde{K}$ chips in $\mathbf{r}(m)$ after sampling the signal at the receiver $r(t)$ in chip rate.

$$\mathbf{r}(m) = \mathbf{S}\mathbf{b}(m) + \mathbf{n}(m) \ , \tag{5}$$

The number of symbols to be considered for one shot depends on $\tilde{K}$ and the starting point of the observation window. Let $\mathbf{s}_u^{(\tilde{m})}$ denote a vector containing the u-th user signature of the $(m+\tilde{m})$-th symbol, i.e. if $\tilde{K}$ is limited to $T_s < \tilde{K}T_c < 3T_s - T_{u,h}$ and the starting point is $mT_s - (K - L_{u,h} + 1)T_c$, then

$$\mathbf{s}_u^{(0)} = \mathbf{s}_u = [0, \cdots, 0, s_u(T_c), \cdots, s_u((K + L_{u,h} - 1)T_c), 0, \cdots, 0]^T$$

$$\mathbf{s}_u^{(-1)} = [s_u((K - L_{u,h} + 1)T_c), \cdots, s_u(KT_c), 0, \cdots, 0]^T$$

$$\mathbf{s}_u^{(+1)} = [0, \cdots, 0, s_u(T_c), s_u(2T_c), \cdots, s_u(L_{u,h} - 1)]^T.$$

In this case $\mathbf{S}$ and $\mathbf{b}(m)$ are given by

$$\mathbf{S} = [\mathbf{s}_1^{(-1)}, \mathbf{s}_1, \mathbf{s}_1^{(+1)}, \bar{\mathbf{s}}_2^{(-1)}, \mathbf{s}_2, \cdots, \mathbf{s}_U^{(-1)}, \mathbf{s}_U, \mathbf{s}_U^{(+1)}]$$

$$\mathbf{b}(m) = [b_1(m - 1), b_1(m), \cdots, b_U(m - 1), b_U(m), b_U(m + 1)]^T$$

The vector $\mathbf{n}(m)$ contains $\tilde{K}$ samples of independent additive white gaussian noise with variance σ_n^2. The convolution of CIR and spreading code after chip rate sampling can also be expressed by the multiplication

$$\mathbf{s}_u = \mathbf{C}_u\mathbf{h}_u = [0, \cdots, 0, (\bar{\mathbf{C}}_u\bar{\mathbf{h}}_u)^T, 0, \cdots, 0]^T \ , \tag{6}$$

where $\bar{\mathbf{C}}_u$ is a $((K+L_{u,h}-1) \times L_{u,h})$-matrix defined by

$$\bar{\mathbf{C}}_p = \begin{bmatrix} c_{u,1} & 0 & \cdots & 0 \\ c_{u,2} & c_{u,1} & & \vdots \\ \vdots & c_{u,2} & \ddots & 0 \\ c_{u,K} & \vdots & & c_{u,1} \\ 0 & c_{u,K} & & c_{u,2} \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & c_{u,K} \end{bmatrix} \tag{7}$$

and $\bar{\mathbf{h}}_u = [h_u(T_c), h_u(2T_c), \cdots, h_u(L_{u,h}T_c]^T$ is a vector of dimension $L_{u,h}$ containing the discrete CIR sampled at chip rate.

For further discussion, we assume a common pilot data sequence $b_i(m) \in [+1, -1]$ of length M, which may be transmitted by an extra pilot channel (where i is the index for the pilot channel) or is an inherent part of a user slot (where i is equal to the user index u). Furthermore, the codes of the interferers are unknown at the receiver. This scenario is typical for CDMA in downlink mode. Nevertheless, all suggested methods can also be applied for the uplink.

3 Conventional Correlative Channel Estimation

The conventional correlative channel estimation can be realized by a RAKE-like structure depicted in **Figure 1**.

Figure 1: Conventional correlative channel estimation

In the first stage, the signal in each path of the shift register is correlated with the spreading sequence, i.e. it is multiplied by the complex conjugate code coefficients $c_{i,k}$ and summed over K chips. The resulting signal is available in symbol rate.

In the second stage, the signals of each path are averaged, resulting in an estimation of discrete CIR, where each path in the register delivers an estimate of the matching channel coefficient.

As only short codes are considered, the spreading sequence $c_i(t)$ is the same for each symbol and the conventional correlative channel estimation can be written as

$$\hat{\mathbf{h}}_i = \frac{1}{M}\mathbf{C}_i^H \sum_{m=1}^{M} b_i(m)\mathbf{r}(m). \tag{8}$$

If we assume a stationary system, the second stage corresponds for $M \to \infty$ to the expectation

$$\lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} b_i(m)\mathbf{r}(m) = E\{b_i(m)\mathbf{r}(m)\} . \qquad (9)$$

Now we can show that the conventional correlative channel estimation is biased. Assume that $b_i(m)$ is uncorrelated to all $b_u(n)$ for $i \neq u$ or $n \neq m$. The expectation term in (9) can be determined by

$$E\{b_i(m)\mathbf{r}(m)\} = \mathbf{s}_i = \mathbf{C}_i\mathbf{h}_i . \qquad (10)$$

Considering (10) we get the expectation of the estimated channel impulse response by

$$E\{\hat{\mathbf{h}}_i\} = \mathbf{C}_i^H\mathbf{C}_i\mathbf{h}_i , \qquad (11)$$

which only conforms to the true CIR if $\mathbf{C}_i^H\mathbf{C}_i$ is an identity matrix. In practice this is generally not the case.

4 Improved Channel Estimation

4.1 Unbiased Correlative Channel Estimation

To obtain an unbiased estimation algorithm, the distance between the received signal vector $\mathbf{r}(m)$ and a reconstructed signature $\mathbf{s}_i$ from user i is minimized according to the MMSE criterion [9]. The target function can be expressed by

$$\begin{aligned} \hat{\mathbf{h}}_i &= \arg\min_{\mathbf{h}_i} f_{tar}(\mathbf{h}_i) \qquad (12) \\ &= \arg\min_{\mathbf{h}_i} E\left\{\|\mathbf{r}(m) - \mathbf{C}_i\mathbf{h}_i b_i(m)\|_2^2\right\} . \end{aligned}$$

$f_{tar}(\mathbf{h}_i)$ is a convex function with an unique minimum which can be determined by setting the partial derivative $\partial f_{tar}(\mathbf{h}_i)/\partial \mathbf{h}_i^H$ to zero:

$$\frac{\partial f_{tar}(\mathbf{h}_i)}{\partial \mathbf{h}_i^H} = \mathbf{C}_i^H\mathbf{C}_i\mathbf{h}_i - \mathbf{C}_i^H E\{\mathbf{r}(m)b_i(m)\} = 0 . \qquad (13)$$

A solution is obtained by replacing the expectation term in (13) by a temporal average as given in (9).

$$\hat{\mathbf{h}}_i = \frac{1}{M}(\mathbf{C}_i^H\mathbf{C}_i)^{-1}\mathbf{C}_i^H \sum_{m=1}^{M} b_i(m)\mathbf{r}(m) \qquad (14)$$

Compared to the conventional correlative channel estimation, (8) incorporates the term $(\mathbf{C}_i^H\mathbf{C}_i)^{-1}$ that can be interpreted as a pre-whitening filter concerning path cross talk. With (10), it can be easily shown that this modification yields an unbiased estimation algorithm. Nevertheless, the variance of this estimation is not protected against MUI.

4.2 Semiblind Channel Estimation

A well-known fact is [2] that the linear single user MMSE-detector can be expressed by

$$\mathbf{d}_{MMSE,u} = q_u\mathbf{\Phi}_{rr}^{-1}\mathbf{s}_u = q_u\mathbf{\Phi}_{rr}^{-1}\mathbf{C}_u\hat{\mathbf{h}}_u , \qquad (15)$$

where q_u is a real positive factor with

$$q_u = \frac{1}{\mathbf{s}_u^H\mathbf{\Phi}_{rr}^{-1}\mathbf{s}_u} \qquad (16)$$

and $\mathbf{\Phi}_{rr}$ is the covariance matrix given by

$$\mathbf{\Phi}_{rr} = E\{\mathbf{r}(m)\mathbf{r}^H(m)\} . \qquad (17)$$

To exactly determine $\mathbf{\Phi}_{rr}$, the knowledge of all user signatures is necessary. If this is not available, we can estimate the covariance matrix by temporal averaging.

$$\mathbf{\Phi}_{rr} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{r}(m)\mathbf{r}^H(m) \qquad (18)$$

In order to avoid the calculation of $\mathbf{\Phi}_{rr}^{-1}$ in (15), the well-known RLS-algorithm (Recursive Least Squares) can be applied for estimating the inverse covariance matrix [10, 11]. A new target function for channel estimation implying an MMSE-detection is given by

$$\begin{aligned} \hat{\mathbf{h}}_i &= \arg\min_{\mathbf{h}_i} f_{tar}(\mathbf{h}_i) \qquad (19) \\ &= \arg\min_{\mathbf{h}_i} E\left\{\|\mathbf{d}_{MMSE}^H\mathbf{r}(m) - b_i(m)\|_2^2\right\} \\ &= \arg\min_{\mathbf{h}_i} E\left\{\|q_i\mathbf{h}_i^H\mathbf{C}_i^H\mathbf{\Phi}_{rr}^{-1}\mathbf{r}(m) - b_i(m)\|_2^2\right\} . \end{aligned}$$

For further derivations we use the substitution $\bar{\mathbf{h}}_i = q_i\mathbf{h}_i$. The unique minimum of $f_{tar}(\bar{\mathbf{h}}_i)$ can be determined by setting the partial derivative $\partial f_{tar}/(\hat{\mathbf{h}}_i)\partial\bar{\mathbf{h}}_i$ to zero:

$$\begin{aligned} \frac{\partial f_{tar}(\bar{\mathbf{h}}_i)}{\partial \bar{\mathbf{h}}_i^H} &= 0 \qquad (20) \\ &= \mathbf{C}_i^H\mathbf{\Phi}_{rr}^{-1}\mathbf{C}_i\hat{\mathbf{h}}_i - \mathbf{C}_i^H\mathbf{\Phi}_{rr}^{-1}E\{\mathbf{r}(m)b_i(m)\} . \end{aligned}$$

A practical solution is obtained by replacing the expectation term in (20) by the temporal average as given in (9).

$$\bar{\mathbf{h}}_i = \frac{1}{M}(\mathbf{C}_i^H\mathbf{\Phi}_{rr}^{-1}\mathbf{C}_i)^{-1}\mathbf{C}_i^H\mathbf{\Phi}_{rr}^{-1}\sum_{m=1}^{M} b_i(m)\mathbf{r}(m) . \qquad (21)$$

Note that no pilot sequence is necessary for calculating the covariance-matrix. Therefore this modification is termed semiblind channel estimation [9]. In contrast to (14), where the reconstructed signature $\mathbf{s}_i$ is compared with the received signal $\mathbf{r}(m)$, the pilot symbols $b_i(m)$ are now directly compared with the linearly processed vector $\mathbf{r}(m)$. The result of (21) can be used for a MMSE-detector. If q_i and $\mathbf{h}_i$ are required, q_i can be calculated by

$$q_i = \mathbf{h}_i^H\mathbf{C}_i^H\mathbf{\Phi}_{rr}^{-1}\mathbf{C}_i\bar{\mathbf{h}}_i . \qquad (22)$$

5 Simulation Results

The performance of the channel estimation algorithms was evaluated for a DS-CDMA system in downlink mode. For spreading we used Hadarmard-codes scrambled with a common pseudo noise code. The spreading factor was $K = 32$ and the transmission was characterized by a 4-tap Rayleigh-fading channel. We assumed a constant channel impulse response for 100 symbols so that we could average over this sequence. The training data stream was transmitted over an extra pilot channel. To investigate the influence of the size of the sliding observation window, we used two different sizes as shown in **Table 1**. Window 1 was proportioned as large as possible, to avoid any distortion caused by inter symbol interference. Window 2 additionally considered one past and one future symbol.

	Window 1
$\tilde{K}$	$K - L_{uh} + 1$
Visible Symbols	1
Starting Point	$mT_s + (L_{uh} - 1)T_c$
	Window 2
$\tilde{K}$	$3K - L_{ph} + 1$
Visible Symbols	3
Starting Point	$mT_s - (K + L_{uh} - 1)T_c$

Table 1: Sliding Observation Window

The results were averaged over 5000 Monte Carlo iterations.

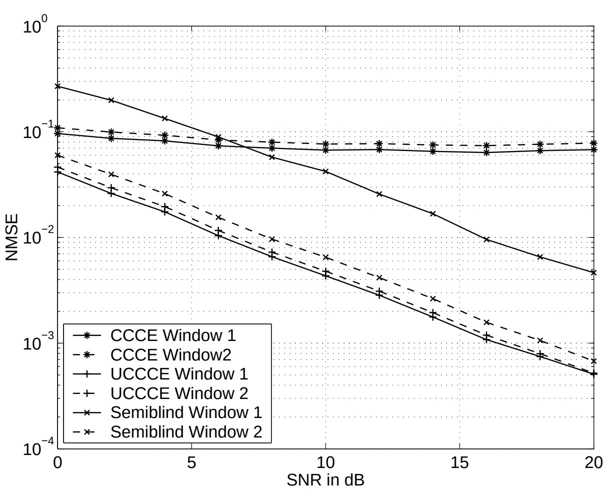

Figure 2: NMSE vs. SNR for a CDMA system with 1 active user

Figure 2 shows the normalized mean squared error (NMSE) between the true and estimated discrete channel impulse response defined by

$$\frac{\|\mathbf{h}_i - \hat{\mathbf{h}}_i\|_2^2}{\|\mathbf{h}_i\|_2^2} \tag{23}$$

versus SNR in dB for a system with only one active pilot data channel. Note that there is no MUI in this scenario. The curve for the conventional correlative channel estimation (CCCE) converges to an error floor biased due to cross talk. In contrast, the results of both improved approaches show no error floor. While the performance of CCCE and unbiased correlative channel estimation (UCCE) slightly increases for window 2, the semiblind channel estimation takes advantage of the small window. The reason for this is that for the semiblind channel estimation we must consider also the variance caused by the estimation of the covariance matrix. This effect is also visible in the following simulations.

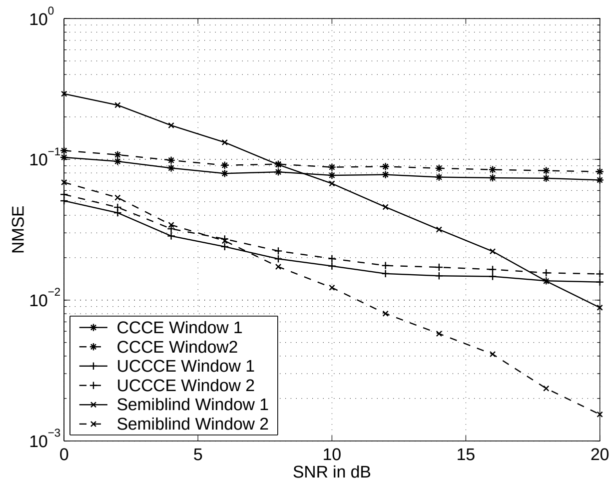

Figure 3: NMSE vs. SNR for a CDMA system with 15 active users and one pilot channel

Figure 3 plots the NMSE values versus SNR for a CDMA system as specified above with 15 active users and one pilot data channel where all signals have the same transmission power. Now the transmission is disturbed by MUI. At about 10 dB the unbiased estimation converges to an error floor due to MUI. The results of semiblind channel estimation still show a linear decreasing curve.

Finally we investigated a CDMA system under extreme near-far conditions. The pilot data channel was disturbed by four users emitting with tenfold power and one user emitting with hundredfold power. As shown in **Figure 4** only the semiblind algorithm has turned out satisfactory.

6 Conclusion

We examined the conventional correlative channel estimation for short-code DS-CDMA systems. Due to the fact that this method delivers biased estimates, we proposed a modification without this drawback. There still remains the drawback

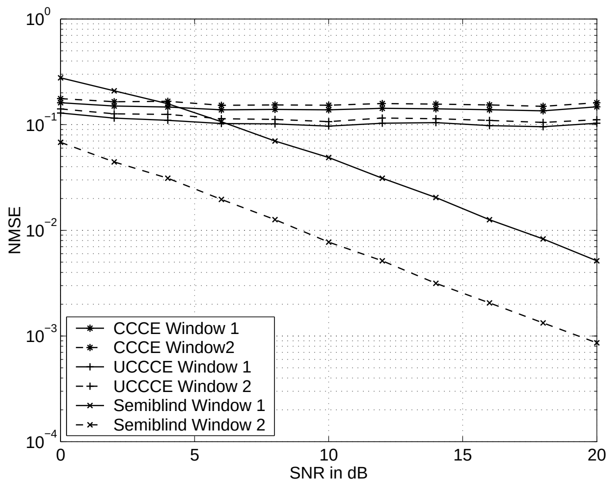

Figure 4: NMSE vs. SNR for a CDMA system under extreme near-far conditions with 6 active user

of a large MUI sensitivity. Therefore we suggested a second modification called semiblind channel estimation. This method promises an improvement to current approaches because it implies a suppression of MUI. The properties of the discussed algorithm could be verified by Monte-Carlo simulations. It is shown that a significant improvement is attained by applying the proposed modifications.

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