Arbeitsbereich Nachrichten Technik

Comparison of Blind Source Separation Methods based on Iterative Algorithms

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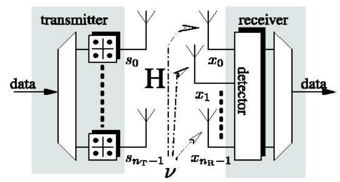
System Setup / Problem Statement

- separation of data streams that are transmitted non-cooperatively
- 200 QPSK symbols
- 4 transmit and 4 receive antennas
- block fading channel

$$\mathbf{x} = \mathbf{H}\cdot\mathbf{s} + \nu$$



MIMO Constant Modulus Algorithms

- general CMA approach

$$J_{\mathrm{CMA}}(\mathbf{C}) = \sum_{t=0}^{n_T-1} \mathrm{E}\left\{\left(|e_t|^2-1\right)^2\right\} = \sum_{t=0}^{n_T-1} \mathrm{E}\left\{\left(\left|\mathbf{c}_t^H\mathbf{x}\right|^2-1\right)^2\right\}$$

MIMO cost function

$$\mathbf{e} = \mathbf{C}^H\cdot\mathbf{x}$$

$$\mathbf{C}^{(i+1)} = \mathbf{C}^{(i)} - \mu\frac{\partial}{\partial\mathbf{C}^{(i)}}J_{\mathrm{CMA}}\left(\mathbf{C}^{(i)}\right)$$ solution with gradient descent

update equation

$$\mathbf{C}^{(i+1)} = \mathbf{C}^{(i)} - 4\mu\mathrm{E}\left\{\left(D\left\{\mathbf{e}^{(i)}\right\}-\mathbf{I}\right)\cdot\mathbf{x}\cdot\mathbf{e}^{(i)H}\right\}$$

$$D\left\{\mathbf{e}^{(i)}\right\} = \mathrm{diag}\left(\left|e_0^{(i)}\right|^2, \left|e_1^{(i)}\right|^2, \ldots, \left|e_{n_T-1}^{(i)}\right|^2\right)$$

problem: separation of different signals can not be guaranteed

1st approach: correlation penalty (CMA corr)

- idea: modify the CMA cost function so that correlated output signals increase its value.

$$J_{\mathrm{corr}}(\mathbf{C}) = J_{\mathrm{CMA}}(\mathbf{C}) + \sum_{k,l=0;\,k\neq l}^{n_T-1}\left|\psi_{k,l}^{(i)}\right|^2$$

$$\psi_{k,l}^{(i)} = \mathrm{E}\left\{e_k^{(i)}\cdot e_l^{(i)*}\right\}$$

update equation

$$\mathbf{C}^{(i+1)} = \mathbf{C}^{(i)} - \mu\left(4\mathrm{E}\left\{\left(D\left\{\mathbf{e}^{(i)}\right\}-\mathbf{I}+\boldsymbol{\Psi}_{\mathrm{corr}}^{(i)\,H}\right)\cdot\mathbf{x}\cdot\mathbf{e}^{(i)H}\right\}\right)$$

$$\boldsymbol{\Psi}_{\mathrm{corr}}^{(i)} = \begin{pmatrix} 0 & \psi_{0,1}^{(i)} & \cdots & \psi_{0,n_T-1}^{(i)} \\ \psi_{1,0}^{(i)} & 0 & \cdots & \psi_{1,n_T-1}^{(i)} \\ \vdots & & \ddots & \vdots \\ \psi_{n_T-1,0}^{(i)} & \psi_{n_T-1,1}^{(i)} & \cdots & 0 \end{pmatrix}.$$

2nd approach: determinant penalty (CMA det)

- idea: modify the CMA cost function so that linear dependent column vetors in MATRIX C increase the cost.

$$J_{\det}(\mathbf{C}) = J_{\mathrm{CMA}}(\mathbf{C}) - \ln\left|\det\mathbf{C}^H\right|$$

update equation

$$\mathbf{C}^{(i+1)} = \mathbf{C}^{(i)} - \mu\left(4\mathrm{E}\left\{\left(D\left\{\mathbf{e}^{(i)}\right\}-1\right)\cdot\mathbf{x}\cdot\mathbf{e}^{(i)H}\right\} - \left(\mathbf{C}^{(i)H}\right)^{-1}\right)$$

3rd approach: subspace limitation (CMA sub)

- idea: separate the signals one by one and ensure orthogonality at initialization

1st step:

$$\mathbf{z} = \mathbf{W}\mathbf{x}$$ whitening (spatial decorrelation)

2nd step:

$$\mathbf{B}_t = [\mathbf{b}_0, \mathbf{b}_1, \ldots \mathbf{b}_{t-1}]$$ start: matrix with zeros

iteration $t = 0\ldots n_T-1$ (all signals)

initialize $\mathbf{b}_t$ with $[0\ldots0\ \underbrace{1}_{t^{\mathrm{th}}\mathrm{entry}}\ 0\ldots0]^{\mathrm{H}}$

$$\mathbf{b}'_t = \mathbf{b}_t - \mathbf{B}_t\mathbf{B}_t^H\mathbf{b}_t$$

solution for one component: steepest descent, CMA cost function

$$e_t = \mathbf{b}_t^{\mathrm{H}}\mathbf{z}$$

$$\mathbf{b}_t^{(i+1)} = \mathbf{b}_t^{(i)} - 4\mu\mathrm{E}\left\{\left(\left|e_t^{(i)}\right|^2-1\right)\cdot\mathbf{z}\cdot e_t^{(i)H}\right\}$$

$$\mathbf{b}''_t = \mathbf{b}'_t/\,||\mathbf{b}'_t||$$

$$\mathbf{e} = \mathbf{B}^H\cdot\mathbf{z}$$

HOS based BSS Approaches

JADE: joint separation of multiple signals by kurtosis maximization (JADE)

1st step:

$$\mathbf{z} = \mathbf{W}\mathbf{x}$$ whitening (spatial decorrelation)

2nd step:

$$\max_{\mathbf{B}} \stackrel{!}{=} \sum_{i,j,l=0}^{n_T-1}\left|\mathrm{cum}\left(e_i^*, e_i, e_j^*, e_l,\right)\right|^2$$ independence (4th order approx.) solved via EVD and joint diagonalization

$$\mathbf{e} = \mathbf{B}^H\cdot\mathbf{z}$$

fastICA: extract of one signal and orthogonalize (fastICA)

1st step:

$$\mathbf{z} = \mathbf{W}\mathbf{x}$$ whitening (spatial decorrelation)

2nd step:

$$\mathbf{B}_t = [\mathbf{b}_0, \mathbf{b}_1, \ldots \mathbf{b}_{t-1}]$$ start: matrix with zeros

iteration $t = 0\ldots n_T-1$ (all signals)

initialize $\mathbf{b}_t$ with random values

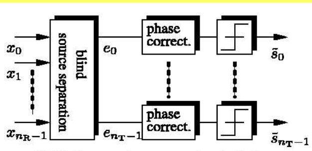
$$e_t = \mathbf{b}_t^{\mathrm{H}}\mathbf{z}$$

solution for one component: fixed point iteration, with constraints

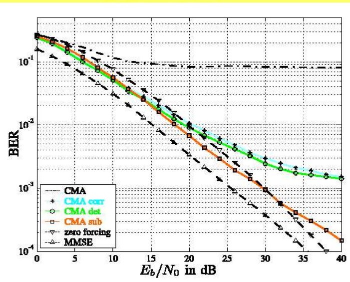
$$\max_{\mathbf{b}_t} J_{\mathrm{fastICA},t}(e_t) = \max_{\mathbf{b}_t}\mathrm{kurt}\{e_t\} = \max_{\mathbf{b}_t}\mathrm{kurt}\left\{\mathbf{b}_t^H\mathbf{z}\right\}$$

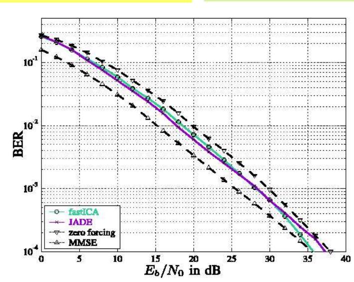
$$\mathbf{b}'_t = \mathbf{b}_t - \mathbf{B}_t\mathbf{B}_t^H\mathbf{b}_t,\quad \mathbf{b}''_t = \mathbf{b}'_t/\,||\mathbf{b}'_t||$$

$$\mathbf{e} = \mathbf{B}^H\cdot\mathbf{z}$$



- all blind separation approaches include a phase and permutation ambiguity problem
- therefore:
  - blind phase correction and symbol decision $e_{t,\mathrm{derot}} = e_t\cdot e^{-j\arg\left(-\mathrm{E}\left\{e_t^4\right\}\right)/4}$
  - for quadrant ambiguity and permutation: utilize ideally known transmit data



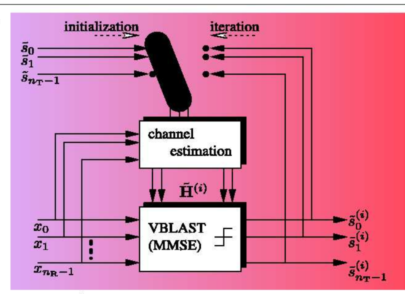


- mandatory for all CMA approaches: strategy to avoid separation of same signals
- new CMA with subspace limitation closely related to fastICA but with CMA cost function
  - outperforms other CMA approaches
  - simple iterative loop: only CMA update
- all blind approaches achieve the performance region of spatial linear filters

Application of Iteration Techniques (Decision Feedback)

motivations for iterative processing:

- BSS-only approach designs a *linear* spatial filter with very low signal knowledge => bad performance, because finite symbol alphabet is not used
- VBLAST: powerful detection algorithm for MIMO-diversity transmissions
- successive interference cancellation
- turbo principle: iteration between data decisions and channel estimations
- free running turbo/decision loop (overall algorithm remains blind)
- problem? After applying BSS and deciding symbols, we get separated signals that are permuted by $\mathbf{P}$ and rotated by $\boldsymbol{\Phi}$. $\hat{\mathbf{S}} = \boldsymbol{\Phi}\mathbf{P}\mathbf{S}$
- What is the estimation result when applying rotated and permuted symbols for channel estimation?
- Assumption: transmission model without noise $\mathbf{X} = \mathbf{H}\mathbf{S}$



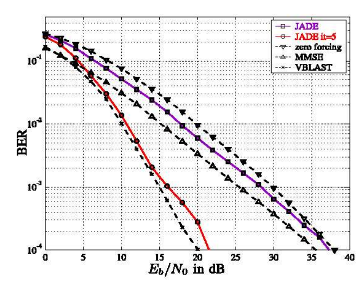
$$\tilde{\mathbf{H}} = \mathbf{X}\cdot\hat{\mathbf{S}}^+ = \mathbf{X}\cdot\hat{\mathbf{S}}^{\mathrm{H}}\left(\hat{\mathbf{S}}\cdot\hat{\mathbf{S}}^{\mathrm{H}}\right)^{-1}$$ estimation with pseudo inverse

$$\tilde{\mathbf{H}} = \mathbf{H}\underbrace{\mathbf{S}\cdot\mathbf{S}^{\mathrm{H}}}_{\approx\sigma_s^2\mathbf{I}}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Phi}^{\mathrm{H}}\left(\boldsymbol{\Phi}\mathbf{P}\underbrace{\mathbf{S}\cdot\mathbf{S}^{\mathrm{H}}}_{\approx\sigma_s^2\mathbf{I}}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Phi}^{\mathrm{H}}\right)^{-1}$$ uncorrelated signals

$$= \mathbf{H}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Phi}^{\mathrm{H}}\left(\boldsymbol{\Phi}\underbrace{\mathbf{P}\mathbf{P}^{\mathrm{H}}}_{=\mathbf{I}}\boldsymbol{\Phi}^{\mathrm{H}}\right)^{-1}$$

$$= \mathbf{H}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Phi}^{\mathrm{H}}\left(\underbrace{\boldsymbol{\Phi}\boldsymbol{\Phi}^{\mathrm{H}}}_{=\mathbf{I}}\right)^{-1}$$

$$\tilde{\mathbf{H}} = \mathbf{H}\mathbf{P}^{\mathrm{H}}\boldsymbol{\Phi}^{\mathrm{H}}$$ permuted and rotated channel matrix



- gain of 10 dB at 10^{-3} (uncoded) by iterative estimation/detection
- gain also shown in real transmissions: J.Rinas and K.D. Kammeyer: "MIMO Measurements of Communication Signals and Application of Blind Source Separation", in *IEEE Symposium on Signal Processing and Information Technology (ISSPIT 2003)*, Darmstadt, Germany, December 2003