Adaptive Distributed MIMO Multi-hop Networks with Optimized Power and Time Allocation

Dirk Wübben
Department of Communications Engineering, University of Bremen
Otto-Hahn-Allee, 28359 Bremen, Germany
Email: wuebben@ant.uni-bremen.de, Phone: +49 421 218 62385

***Abstract*—Distributed multiple-input multiple-output (MIMO) multi-hop relaying is a promising technology to achieve high-quality end-to-end (e2e) link performance by providing co-operative diversity. The application of low-complexity space-time codes offers improved coverage, data rates, and reliability. However, efficient transmission protocols and enhanced resource allocation schemes are required to exploit these advantages. Specifically, low-complexity adaptive schemes and power control strategies should be designed, thereby achieving robust and cost-efficient e2e communications. In this paper an adaptive multi-hop transmission scheme is presented, where one relay stops forwarding the message if it is in outage and the remaining relays adapt to a new space-time code. For this adaptive scheme the e2e outage probability is derived and the optimum resource allocation strategy that reduces the total transmit power while satisfying the given e2e Quality-of-Services (QoS) requirement is presented. In addition to this optimum joint power and time allocation a near-optimal closed-form solution based on several approximations is derived. By simulation results significant power savings for the adaptive approach in comparison to a non-adaptive scheme and the quality of the near-optimum solution are demonstrated.**

***Index Terms*—Relaying, Distributed MIMO, Virtual Antenna Arrays, Joint Power and Time Allocation.**

I. Introduction

Recently, there has been increasing interest in applying traditional point-to-point MIMO techniques into multi-hop wireless relaying networks to support higher e2e data rates and to provide a better user experience [1]–[4]. By the concept of virtual antenna array (VAA) spatially separated relaying nodes can utilize the capacity improvements offered by MIMO transmission techniques. For example, the application of distributed space-time codes was proven in [5] to significantly improve the data rate in multi-hop networks. Fig. 1 depicts a distributed MIMO multi-hop network, where one source communicates with one destination via a number of relaying VAAs in multiple hops. Spatially adjacent nodes in a VAA receive data from the previous VAA and relay data to the consecutive VAA until the destination is reached.

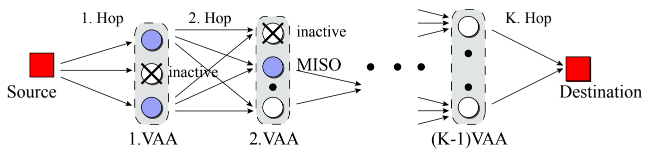

Fig. 1. Topology of adaptive distributed MIMO multi-hop relaying systems.

The general concept of distributed MIMO multi-hop communication systems has been analyzed in [5], where explicit resource allocation strategies were introduced to maximize the e2e data throughput over ergodic fading channels. In contrast, we consider the non-ergodic outage probability which is applicable for the majority of real-world wireless applications [6]–[11]. As discussed in [2], the drawback of the fixed decode-and-forward transmission is that it requires full decoding at all relays. Thus, the e2e connection is considered to be in outage if any relay can not decode the message correctly. The e2e performance is then determined by the worst relay link in the network. A similar assumption has also been made in [6]–[8] in terms of e2e outage probability and in [4] in terms of ergodic capacity. This strong assumption degrades the e2e performance drastically. Thus, a simple adaptive decode-and-forward scheme for distributed MIMO multi-hop networks will be considered here.

The aim of this paper is to present and investigate resource-allocation strategies for e2e outage probability constrained adaptive transmissions over slow-fading channels. This is achieved by optimally assigning resources in terms of fractional time and transmission power to the hops. Based on the work [9], [10] the joint power and time allocation problem is formulated as a convex optimization problem, which can be solved by common optimization tools with considerable complexity. To reduce complexity, a sub-optimal but efficient solution will be derived. The power savings in comparison to a non-adaptive scheme [7] and to an adaptive scheme with pure power optimization as proposed in [11] will be demonstrated.

The remainder of the paper is organized as follows. In Section II the system model of the adaptive transmission scheme is introduced. The mathematical description of the outage probability will be given in Section III and the optimal power allocation problem is formulated as a convex optimization problem in Section IV. A closed-form solution for an approximated optimization problem will be derived in Section V. Finally, performance results and conclusions will be given in Section VI and VII, respectively.

II. System description

We consider a K-hop network with t_k transmit nodes and r_k receive nodes at hop k, i.e. $t_{k+1} = r_k$. Several relays are grouped to a VAA at each hop to apply a distributed space-time code. The data is then transmitted from the source to the destination by $K-1$ VAAs. It is assumed that no interference between the hops occur. Thus, the bandwidth or time has to be divided into non-overlapping parts for each hop such that at any time they are occupied by only one hop, i.e., FDMA or TDMA respectively. Without loss of generality, the TDMA based adaptive scheme will be considered here.

At the first time fraction α_1 the source transmits data to the relays of the 1. VAA. The nodes of the 1. VAA decode the received signals separately in order to avoid enormous information exchanges, i.e., *separately decoding* is performed, which decomposes this hop to several SISO links (or MISO links at the next hops). The t'_k relays successfully decoding the message (or being not in outage) are denoted as *active nodes* and the others failing to decode the message (or being in outage) are denoted as *inactive nodes*, respectively. The inactive nodes will stop transmission at the next time fraction. The t'_k active nodes will *adapt* to transmit the decoded message *cooperatively* according to a space-time code with respect to t'_k, i.e., $0 \leq t'_k \leq t_k$. To this end, each node transmits a spatial fraction of a space-time code. If all relays within one VAA fail to decode the message, the e2e connection is considered to be in outage, denoted by the probability P_{e2e}. Otherwise, the t'_k active nodes send the data to the next VAA at the time fraction α_k. This adaptive transmission continues at each VAA until the message reaches the destination.

It is assumed that each relay transmits signals with the same data rate R but with individual time fraction α_k, of which $\sum_{k=1}^{K}\alpha_k = 1$ holds. All the hops use the total bandwidth W that is available to the network. We define $\mathbf{S}_k \in \mathbb{C}^{t'_k \times L_k}$ as the space-time encoded signal with length L_k from the t'_k active nodes at the kth hop. The received signal $\mathbf{y}_{k,j} \in \mathbb{C}^{1\times L_k}$ at the jth node at the kth VAA is given by

$$\mathbf{y}_{k,j} = \sqrt{\theta_k \mathcal{P}_k / t_k}\mathbf{h}_{k,j}\mathbf{S}_k + \mathbf{n}_{k,j} \ , \quad (1)$$

where $\mathbf{n}_{k,j} \sim \mathcal{N}_C(0, N_0) \in \mathbb{C}^{1\times L_k}$ is the Gaussian noise vector with power spectral density N_0. Each active node from one VAA transmits data with power $\mathcal{P}_k/t_k$ equally. This permits simple power control and hardware implementation at each relay which is especially important for relaying nodes with minimal processing functionality. The channel from the t'_k active nodes to the jth receive node within the kth hop is denoted as $\mathbf{h}_{k,j} \in \mathbb{C}^{1\times t'_k}$, whose elements obey the same uncorrelated Rayleigh fading statistics with unit variance. It is assumed that the relaying nodes belonging to the same VAA are spatially sufficiently close as to justify a common path loss θ_k between two VAAs, which is known as *symmetric network*. The path loss is described as $\theta_k = d_k^{-\epsilon}$, where d_k is the distance between two nodes and ϵ is the path loss exponent within range of 2 to 5 for most wireless channels.

In order to meet a given Quality-of-Services (QoS) requirement, the transmit power $\mathcal{P}_k$ and the time fraction α_k per hop need to be optimized. In the next section the e2e outage probability is introduced as the QoS parameter and optimum as well as near-optimum solutions to the occurring optimization problem are proposed subsequently.

III. End-to-End Outage Probability

Before formulating the outage probability $P_{\text{out},k}$ of hop k, we first consider the outage probability $p_{\text{out},k,j}(t'_k)$ of a MISO system with t'_k active nodes at hop k as described in (1). The instantaneous achievable rate of the $t'_k \times 1$ link is given by

$$C_{k,j}(t'_k) = \alpha_k W \log_2\left(1 + \frac{\mathcal{P}_k}{\alpha_k W t_k d_k^{\epsilon} N_0}\|\mathbf{h}_{k,j}\|^2\right) \ , \quad (2)$$

with $\|\mathbf{h}_{k,j}\|^2 = \sum_{i=1}^{t'_k}|h_{k,j,i}|^2$. The outage probability $p_{\text{out},k,j}(t'_k)$ can be expressed as the probability that the channel can not support an error-free transmission at rate R

$$\begin{aligned} p_{\text{out},k,j}(t'_k) &= \Pr(R > C_{k,j}(t'_k)) \quad (3)\\ &= \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \frac{\left(2^{\frac{R}{\alpha_k W}}-1\right)\alpha_k W N_0 d_k^{\epsilon} t_k}{\mathcal{P}_k}\right) . \end{aligned}$$

Clearly, any analytical optimization of (3) in terms of the fractional time α_k and power $\mathcal{P}_k$ is intractable due to the fairly evolved expression. To overcome that problem, the approximation $\log_2(1+u) \approx \sqrt{u}$ to the achievable rate in (2) has been employed in [5]. Thus, (2) can be simplified by

$$C_{k,j}(t'_k) \approx \sqrt{\frac{\alpha_k W \mathcal{P}_k}{d_k^{\epsilon} N_0 t_k}\|\mathbf{h}_{k,j}\|^2} \quad (4)$$

and the outage probability (3) becomes

$$\begin{aligned} p_{\text{out},k,j}(t'_k) &\approx \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \frac{R^2 N_0 d_k^{\epsilon} t_k}{\alpha_k W \mathcal{P}_k}\right) \quad (5a)\\ &= \Pr\left(\|\mathbf{h}_{k,j}\|^2 < x_k\right) \quad (5b)\\ &= \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} . \quad (5c) \end{aligned}$$

To simplify the notation $x_k = Q_k/(\alpha_k \mathcal{P}_k)$ is used with variable $Q_k = R^2 N_0 d_k^{\epsilon} t_k / W$. In (5b), $\|\mathbf{h}_{k,j}\|^2$ obeys a Gamma distribution [12], therefore its CDF can be described by an incomplete Gamma function $\gamma(t'_k, x_k) = \int_0^{x_k} e^{-u}u^{t'_k-1}\,\mathrm{d}u$ normalized by Gamma function $\Gamma(t'_k)$ as given in (5c). Clearly, the outage probability $p_{\text{out},k,j}(t'_k)$ depends on the number of active t'_k at hop k, whereas the probability of t'_k active nodes is determined by the outage probability of the nodes at the previous hop.

The outage probability of receiving node j at hop k is denoted by $P_{\text{out},k,j}$. Under the assumption of symmetric networks the outage probabilities of the nodes within one VAA are equal, i.e., $P_{\text{out},k,1} = \cdots = P_{\text{out},k,r_k} = P_{\text{out},k,j'}$ where j' indexes an arbitrary $j \in [1, \cdots, r_k]$. The number of active nodes t'_k at hop k is a random number that depends on the outage probabilities in the previous hop $k-1$. As these probabilities $P_{\text{out},k-1,j'}$ are equal, the number of active nodes t'_k follows the binomial distribution $\mathcal{B}$ with parameters t_k and $P_{\text{out},k-1,j'}$ and we write [12]

$$t'_k \sim \mathcal{B}(t_k, 1 - P_{\text{out},k-1,j'}) \ . \quad (6)$$

The probability of i nodes being active at hop k is expressed by the probability mass function as

$$\Pr(t'_k = i) = \binom{t_k}{i}(1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k - i} \ , \ \forall i \quad (7)$$

with $\binom{t_k}{i} = \frac{t_k!}{i!(t_k-i)!}$. As the outage probability of a MISO system with i active nodes is described by $\Pr(t'_k = i) \cdot p_{\text{out},k,j}(i)$, the outage probability $P_{\text{out},k,j'}$ is given by the sum of the outage probabilities over all possible i

$$P_{\text{out},k,j'} = \sum_{i=1}^{t_k} \Pr(t'_k = i) \cdot p_{\text{out},k,j}(i) \tag{8a}$$

$$= \sum_{i=1}^{t_k} \binom{t_k}{i} (1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k - i} \frac{\gamma(i, x_k)}{\Gamma(i)} \; . \tag{8b}$$

If all receive nodes of one hope can not decode the message the corresponding hop is in outage. Thus, the outage probability of hop k is given by

$$P_{\text{out},k} = \prod_{j=1}^{r_k} P_{\text{out},k,j} = P_{\text{out},k,j'}^{r_k} \; . \tag{9}$$

Consequently the e2e connection is in outage if any hop is broken and the e2e outage probability corresponds to

$$P_{\text{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\text{out},k}) = 1 - \prod_{k=1}^{K} \left(1 - P_{\text{out},k,j'}^{r_k}\right) \; . \tag{10}$$

In the following investigation we use the end-to-end outage probability P_{e2e} as the measurement for the required QoS.

IV. Optimum Joint Power and Time Allocation (JPTA)

The joint power and time allocation task for the adaptive scheme is formulated in order to minimize the total power consumption meanwhile supporting a given e2e outage probability requirement e as follows

$$\text{minimize } \mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \mathcal{P}_k (1 - P_{\text{out},k-1,j'}) \tag{11a}$$

$$\text{subject to } P_{\text{e2e}} \le e \quad \text{and} \quad \sum_{k=1}^{K} \alpha_k = 1. \tag{11b}$$

The calculation of $\mathcal{P}_{\text{total}}$ considers the inactive nodes stopping the transmission to save power. Problem (11) can be shown to be convex for low outage probability requirements by proving the Hessian matrix of $P_{\text{e2e}}(\mathcal{P}_k, \alpha_k, \forall k)$ to be positive semi-definite. To this end, the optimal solution $\mathcal{P}_k^\star$, $\alpha_k^\star$ for (11) can be obtained by standard optimization tools leading to considerable complexity [13].

V. Approximated Joint Power and Time Allocation

A. Problem Simplification

To simplify the problem, some approximations to the outage probability are invoked that permit the derivation of a near-optimal closed-form power allocation solution. Following the approximation method given in [6], [14], the outage probability $p_{\text{out},k,j}(t'_k)$ in (5c) is upper bounded for high SNR as

$$p_{\text{out},k,j}(t'_k) = \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} \lessapprox \frac{t_k'^{-1} x_k^{t'_k}}{\Gamma(t'_k)} = \frac{x_k^{t'_k}}{\Gamma(t'_k + 1)} \; . \tag{12}$$

Hence, the outage probability of node j' in hop k defined in (8a) is approximated by $\tilde{P}_{\text{out},k,j'}$

$$P_{\text{out},k,j'} \lessapprox \sum_{i=1}^{t_k} \Pr(t'_k = i) \frac{x_k^i}{\Gamma(i+1)} \triangleq \tilde{P}_{\text{out},k,j'} \; . \tag{13}$$

The end-to-end outage probability (10) can be further approximated by the union bound [6]

$$P_{\text{e2e}} \le \sum_{k=1}^{K} P_{\text{out},k} = \sum_{k=1}^{K} P_{\text{out},k,j'}^{r_k} \le \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \triangleq \tilde{P}_{\text{e2e}} \; . \tag{14}$$

For small $P_{\text{out},k-1,j'}$ the objective function of the optimization problem (11) can be rewritten to $\mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \alpha_k \mathcal{P}_k$. Thus, the approximated optimization problem is obtained

$$\text{minimize } \mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \mathcal{P}_k \tag{15a}$$

$$\text{subject to } \tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \le e \text{ and } \sum_{k=1}^{K} \alpha_k = 1 \; . \tag{15b}$$

B. Closed-Form Solution (JPTA-CF)

The optimization problem (15) only leads to a near-optimal solution. However, from the complexity point of view, it is attractive to use (15) to derive efficient solutions. To solve the problem, the Lagrangian of the approximated optimization problem is defined as

$$L(\alpha_k, \mathcal{P}_k) = \sum_{k=1}^{K} \mathcal{P}_k + \lambda(\tilde{P}_{\text{e2e}} - e) + \nu \left(\sum_{k=1}^{K} \alpha_k - 1 \right) \; . \tag{16}$$

To obtain the sub-optimal solution, the derivatives of $L(\alpha_k, \mathcal{P}_k)$ with respect to α_k and $\mathcal{P}_k$ has to be zero for all $1 \le k \le K$, i.e.,

$$\frac{\partial L(\alpha_k, \mathcal{P}_k)}{\partial \mathcal{P}_k} = 1 + \lambda \frac{\partial \tilde{P}_{\text{e2e}}}{\partial x_k} \frac{\partial x_k}{\partial \mathcal{P}_k} = 1 - \lambda \frac{\partial \tilde{P}_{\text{e2e}}}{\partial x_k} \frac{x_k}{\mathcal{P}_k} = 0 \; , \tag{17a}$$

$$\frac{\partial L(\alpha_k, \mathcal{P}_k)}{\partial \alpha_k} = \lambda \frac{\partial \tilde{P}_{\text{e2e}}}{\partial x_k} \frac{\partial x_k}{\partial \alpha_k} + \nu = -\lambda \frac{\partial \tilde{P}_{\text{e2e}}}{\partial x_k} \frac{x_k}{\alpha_k} + \nu = 0 \; . \tag{17b}$$

By inserting (17b) in (17a) the relation

$$\alpha_k = \frac{\mathcal{P}_k}{\nu} \tag{18}$$

is achieved. Using this relation with the condition $\sum_{k=1}^{K} \alpha_k = 1$ it becomes obvious that the Lagrangian variable ν equals the sum power

$$\nu = \sum_{k=1}^{K} \mathcal{P}_k \tag{19}$$

and the time fraction per hop is consequently given by

$$\alpha_k = \frac{\mathcal{P}_k}{\sum_{k=1}^{K} \mathcal{P}_k} \; . \tag{20}$$

Thus, α_k corresponds to the relative power fraction per hop. Inserting (18) into $x_k = Q_k / (\alpha_k \mathcal{P}_k)$ introduced in (5b), the variable x_k becomes

$$x_k = \frac{Q_k \nu}{\mathcal{P}_k^2} \; . \tag{21}$$

Furthermore, for the optimum solution of (15) the constraint function (15b) must be fulfilled with equality

$$\tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k} = e \,. \tag{22}$$

From (17) and (22), a closed-form solution for α_k and $\mathcal{P}_k$ can be achieved by several further approximations as outlined in the Appendix.

Theorem 1: [Joint Power and Time Allocation in Closed Form (JPTA-CF)] The joint power and time (or bandwidth) allocation for outage restricted adaptive distributed MIMO multi-hop networks in closed form is given by

$$\mathcal{P}_k^\star = \tilde{P}_{\text{out},k,j'}^{\frac{-1}{t_k+1}} A'_k \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{\frac{-1}{t_k+1}} A'_k \qquad \text{and} \tag{23a}$$

$$\alpha_k^\star = \frac{\mathcal{P}_k^\star}{\sum_{k=1}^{K} \mathcal{P}_k^\star} \,, \tag{23b}$$

with approximated outage probability per hop

$$\tilde{P}_{\text{out},k} = \tilde{P}_{\text{out},k,j'}^{r_k} \approx \frac{\delta_k \cdot e}{\sum_{k=1}^{K} \delta_k} \tag{24}$$

and parameters δ_k, A_k and A'_k given by

$$\delta_k = \left(\frac{A_k}{r_k(t_k+1)} \right)^{\frac{(t_k+1)r_k}{1+(t_k+1)r_k}} \tag{25a}$$

$$A_k = t_k^{\frac{1}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k(t_k+1)}} \tag{25b}$$

$$A'_k = t_k^{\frac{1}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1-\tilde{P}_{\text{out},k-1}^{\frac{1}{r_{k-1}}})^i \tilde{P}_{\text{out},k-1}^{\frac{t_k-i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k(t_k+1)}} \tag{25c}$$

VI. Performance Evaluation

In the sequel the performance of several resource allocation schemes are evaluated for two different multi-hop network with $K = 3$ hops. It is assumed that the e2e communication over $W = 5$ MHz should meet an e2e outage probability constraint of $e = 1\%$ with the path loss exponent $\epsilon = 3$ and $N_0 = -174$ dBm/Hz. For comparison, the performance is shown for non-adaptive transmission [6], [7] and adaptive transmissions with optimized resource allocations. Note that the optimal solution *JPTA* (11) is solved by means of standard optimization methods [13] and the closed-form solution *JPTA-CF* is given by Theorem 1. In order to reveal the benefits of joint power and time allocation, the optimal power allocation with equal time for each hop (i.e., $\alpha_k = 1/K, \forall k$) is considered for comparison [11] and is denoted *PA* for brevity.

A. System A

In this section a multi-hop system with $K = 3$ hops, three nodes per VAA and an equal distance between the nodes of $d = 1$ km is investigated.

In Fig. 2 the total power $\mathcal{P}_{\text{total}}$ versus the data rate R is shown for the different allocation schemes. It can be observed

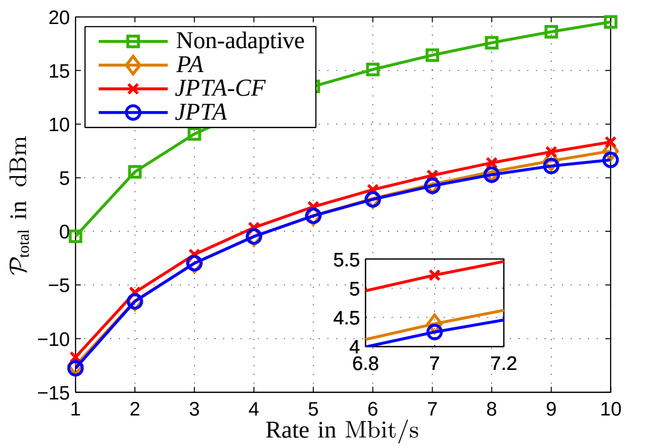

Fig. 2. $\mathcal{P}_{\text{total}}$ in dBm for a) Non-adaptive transmission and adaptive transmission with b) *PA*, c) *JPTA*, and d) *JPTA-CF* for System A.

that the proposed adaptive transmission scheme with optimum resource allocation *JPTA* achieves a gain of approximately $12\,\text{dBm}$ in comparison to the non-adaptive scheme, where the e2e connection is in outage if any node in the network is in outage [6], [7]. The adaptive transmission scheme with pure power optimization *PA* results in almost the same total transmit power as the assumed time allocation $\alpha_k = 1/3$ equals almost the time allocation determined by JPTA. The proposed closed-form solution *JPTA-CF* yields near-optimum total power consumption and results in a power increase of less than $1\,\text{dBm}$ in comparison to the optimum solution *JPTA*, but with a strong reduction in computational complexity.

Fig. 3 depicts the power $\mathcal{P}_k$ of each hop for the four approaches under investigation. For the non-adaptive scheme the first hop uses the most power, which is due to the lack of diversity degrees at the first hop. In contrast, the adaptive schemes consume most energy in the last hop. The reason for this is, that there is only one node at the destination which has to decode the data, correctly otherwise an outage event occurs. Similar, at the source there are no nodes to transmit the data cooperatively with high diversity degrees. Thus, the source consumes the second most power. Due to the adaptive scheme and space-time coding at the second hop, this hop uses the least power. Comparing the adaptive schemes, the equivalence of *PA* and *JPTA* becomes obvious. Both consume approximately the same power per hop to meet the e2e outage requirement P_{e2e}. In general, the adaptive transmission leads to a more balanced dispersion of the power.

B. System B

For the second considered scenario it assumed that the first and the second VAA contain two and five nodes, respectively. Furthermore, the distance between the VAA's is $d_1 = 3$ km and $d_2 = d_3 = 1$ km. Obviously, the link between the source and the first VAA is the weakest and will correspondingly require most of the resources.

The total transmit power for the different allocation schemes is shown in Fig. 4 the total power. The adaptive transmission scheme with pure power optimization *PA* results already in

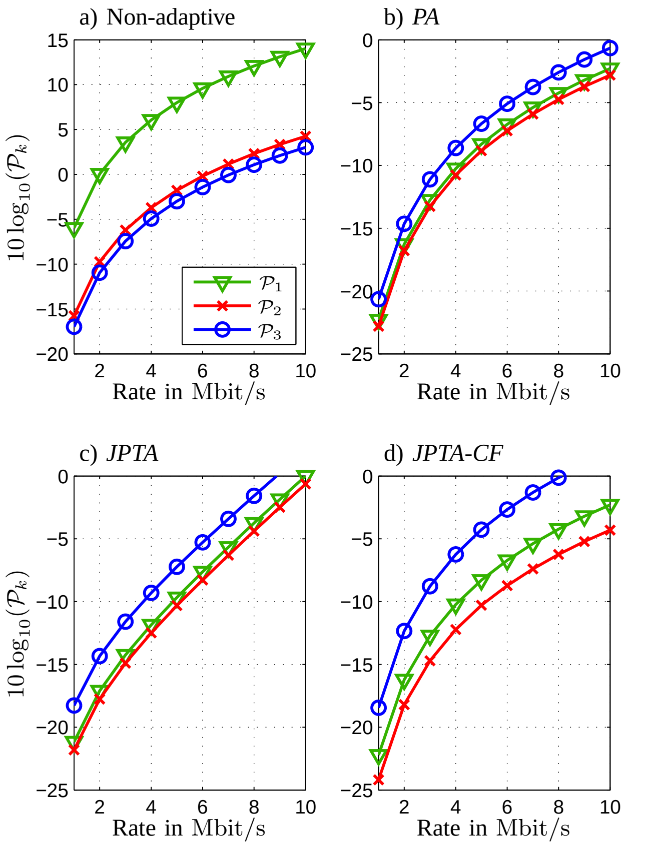

Fig. 3. Power $\mathcal{P}_k$ per hop fora) Non-adaptive transmission and adaptive transmission with b) *PA*, c) *JPTA*, and d) *JPTA-CF* for System A.

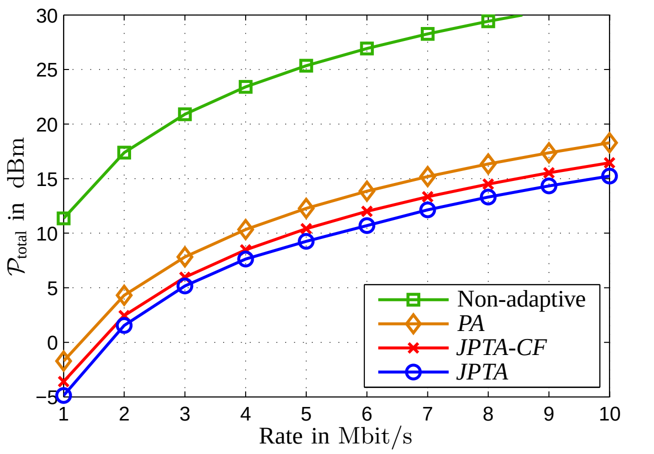

Fig. 4. $\mathcal{P}_{\text{total}}$ in dBm for a) Non-adaptive transmission and adaptive transmission with b) *PA*, c) *JPTA*, and d) *JPTA-CF* for System B.

strong power reductions. However, the loss of approximately $3.2\,\mathrm{dBm}$ with respect to *JPTA* affirms the necessity to allocate both resources power and time jointly in this scenario. The proposed closed-form solution *JPTA-CF* outperforms *PA* and results in a power increase of less than $1\,\mathrm{dBm}$ in comparison to the optimum solution *JPTA*.

VII. Summary and Conclusions

In this paper, we derived a joint power and time allocation approach for adaptive distributed MIMO multi-hop networks with optimal as well as closed-form solutions. As shown by numerical investigations, the adaptive scheme outperforms the non-adaptive scheme significantly and a joint optimization of power and time (or bandwidth) is favorable for systems with nonequal link strength. Furthermore, the closed form joint power and time allocation *JPTA-CF* achieves near-optimal performance with lower complexity comparing to the optimal solution *JPTA*.

VIII. Appendix

Proof of Theorem 1: The first derivative of $L(\alpha_k, \mathcal{P}_k)$ in (17) with respect to $\mathcal{P}_k$ relates to $\tilde{P}_{\text{out},k}$ as well as $\tilde{P}_{\text{out},k+1}$, expressed as follows

$$\frac{\partial L(\alpha_k, \mathcal{P}_k)}{\partial \mathcal{P}_k} = 1 + \lambda \left(\frac{\partial \tilde{P}_{\text{out},k}}{\partial \mathcal{P}_k} + \frac{\partial \tilde{P}_{\text{out},k+1}}{\partial \mathcal{P}_k} \right) = 0\,, \qquad (26)$$

which is due to the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ indicted in (8a). This makes the further analysis involved.

However, by recognizing that $\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} < e$, $\forall\, k$ the dependency in (8a) can be removed by replacing the outage probability $\tilde{P}_{\text{out},k-1,j'}$ by $e^{\frac{1}{r_{k-1}}}$. Thus, (13) becomes

$$\tilde{P}_{\text{out},k,j'} \approx \sum_{i=1}^{t_k} \binom{t_k}{i} \left(1 - e^{\frac{1}{r_{k-1}}}\right)^i e^{\frac{t_k - i}{r_{k-1}}} \frac{x_k^i}{\Gamma(i+1)}\,. \qquad (27)$$

Furthermore, (27) can be approximated by its geometric mean

$$\tilde{P}_{\text{out},k,j'} \approx t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} x_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} \qquad (28)$$

$$= x_k^{\frac{t_k+1}{2}} t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{1}{t_k}}$$

$$= \left(\frac{Q_k \nu}{\mathcal{P}_k^2} \right)^{\frac{t_k+1}{2}} t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{1}{t_k}}$$

Hence, $\mathcal{P}_k$ can be expressed by $\tilde{P}_{\text{out},k,j'}$ as

$$\mathcal{P}_k = \frac{\sqrt{\nu}\, t_k^{\frac{1}{t_k+1}}}{\tilde{P}_{\text{out},k,j'}^{\frac{1}{t_k+1}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k(t_k+1)}}$$

$$= \sqrt{\nu}\, \tilde{P}_{\text{out},k,j'}^{\frac{-1}{t_k+1}} A_k\,, \qquad (29)$$

with parameter

$$A_k = t_k^{\frac{1}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k(t_k+1)}} \qquad (30)$$

used to simplify the notation. As the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ has been removed, equation (26) simplifies to

$$\frac{\partial L(\alpha_k, \mathcal{P}_k)}{\partial \mathcal{P}_k} = 1 + \lambda \frac{\partial \tilde{P}_{\text{out},k}}{\partial \mathcal{P}_k} = 0\,. \qquad (31)$$

Differentiating (28) along $\mathcal{P}_k$ yields

$$0 = 1 + \lambda r_k \tilde{P}^{r_k - 1}_{\text{out},k,j'} \frac{\partial \tilde{P}_{\text{out},k,j'}}{\partial \mathcal{P}_k} \tag{32a}$$

$$= 1 - \frac{\lambda r_k (t_k + 1) \tilde{P}^{r_k - 1}_{\text{out},k,j'}}{\mathcal{P}_k} \tilde{P}_{\text{out},k,j'} \tag{32b}$$

$$= 1 - \frac{\lambda r_k (t_k + 1)}{\mathcal{P}_k} \tilde{P}^{r_k}_{\text{out},k,j'} \tag{32c}$$

$$= 1 - \frac{\lambda r_k (t_k + 1)}{\mathcal{P}_k} \tilde{P}_{\text{out},k} \; . \tag{32d}$$

Inserting (29) in (32d), $\tilde{P}_{\text{out},k}$ is given by

$$\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} = \left(\frac{\lambda}{\sqrt{\nu}} \right)^{-\frac{(t_k+1) r_k}{1+(t_k+1) r_k}} \cdot \delta_k \; , \tag{33}$$

where δ_k is introduced to simply the notation

$$\delta_k = \left(\frac{A_k}{r_k (t_k + 1)} \right)^{\frac{(t_k+1) r_k}{1+(t_k+1) r_k}} \; . \tag{34}$$

Since $(\lambda/\sqrt{\nu})^{-\frac{(t_k+1) r_k}{1+(t_k+1) r_k}}$ can be approximated by $(\lambda/\sqrt{\nu})^{-1}$ for large t_k, inserting (33) in (22) yields

$$\left(\frac{\lambda}{\sqrt{\nu}} \right)^{-1} \approx \frac{e}{\sum_{k=1}^{K} \delta_k} \; . \tag{35}$$

Hence, the approximated outage probability is given by

$$\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} \approx \frac{\delta_k}{\sum_{k=1}^{K} \delta_k} \cdot e \; . \tag{36}$$

Since $\nu = \sum_{k=1}^{K} \mathcal{P}_k$ given in (19), the power for hop k in (29) becomes

$$\mathcal{P}_k = \sqrt{\sum_{k=1}^{K} \mathcal{P}_k} \tilde{P}^{\frac{-1}{t_k+1}}_{\text{out},k,j'} A_k \; , \tag{37}$$

The total power $\mathcal{P}_{\text{total}}$ corresponds to the sum

$$\mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \mathcal{P}_k = \sqrt{\sum_{k=1}^{K} \mathcal{P}_k} \cdot \sum_{k=1}^{K} \tilde{P}^{\frac{-1}{t_k+1}}_{\text{out},k,j'} A_k \tag{38}$$

and leads to the relation between power and outage probability

$$\sqrt{\sum_{k=1}^{K} \mathcal{P}_k} = \sum_{k=1}^{K} \tilde{P}^{\frac{-1}{t_k+1}}_{\text{out},k,j'} A_k \; . \tag{39}$$

Inserting (36) and (39) in (37), and replacing e in A_k by $\tilde{P}_{\text{out},k-1}$, we finally achieve $\mathcal{P}_k$ given in Theorem 1. This concludes the proof. ■

References

[1] J. Laneman and G. Wornell, "Distributed Space-Time-Coded Protocls for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. on Information Theory*, vol. 59, no. 10, pp. 2415–2425, Oct. 2003.

[2] J. Laneman, D. Tse, and G. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Feb. 2004.

[3] L. Le and E. Hossain, "Multihop Cellular Networks: Potential Gains, Research Challenges, and a Resource Allocation Framework," *IEEE Communications Magazine*, vol. 45, pp. 66–73, Sep. 2007.

[4] M. Dohler, A. Gkelias, and A. Aghvami, "A Resource Allocation Strategy for Distributed MIMO Multi-Hop Commonication Systems," *IEEE Communications Letters*, vol. 8, no. 2, pp. 98–101, Feb. 2004.

[5] M. Dohler, "Virtual Antenna Arrays," Ph.D. dissertation, King's College, University of London, London, UK, Nov. 2003.

[6] Y. Lang, D. Wübben, C. Bockelmann, and K.-D. Kammeyer, "A Closed Power Allocation Solution for Outage Restricted Distributed MIMO Multi-hop Networks," in *Proc. Workshop on Resource Allocation in Wireless Networks (RAWNET)*, Berlin, Germany, Mar. 2008.

[7] D. Wübben and Y. Lang, "Near-Optimum Power Allocation Solutions for Outage Restricted Distributed MIMO Multi-Hop Networks," in *IEEE Proc. Global Communications Conference (GLOBECOM)*, New Orleans, USA, Nov. 2008.

[8] Y. Lang, D. Wübben, and K.-D. Kammeyer, "Efficient Power Allocation for Outage Restricted Asymmetric Distributed MIMO Multi-hop Networks," in *IEEE Proc. International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Cannes, France.

[9] Y. Lang and D. Wübben, "Performance Evaluation of Joint Power and Time Allocations for Adaptive Distributed MIMO Multi-hop Networks," in *Proc. ITG Workshop on Smart Antennas*, Berlin, Germany, Feb. 2009.

[10] Y. Lang, D. Wübben, and K.-D. Kammeyer, "Joint Power and Time Allocations for Adaptive Distributed MIMO Multi-hop Networks," in *IEEE Proc. Vehicular Technology Conference (VTC), Spring*, Barcelona, Spain, Apr. 2009.

[11] ——, "Power Allocations for Adaptive Distributed MIMO Multi-hop Networks," in *IEEE Proc. International Conference on Communications (ICC)*, Dresden, Germany, Jun. 2009.

[12] M. Kendall and A. Stuart, *The Advanced Theory of Statistics*, 4th ed. London, U.K.: Griffen, 1979, vol. 1.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[14] M. Dohler and M. Arndt, "Inverse Incomplete Gamma Function and its Application," *IEE Electr. Letters*, vol. 42, no. 1, pp. 35–36, Jan. 2006.