

Efficient Coded Bit and Power Loading for BICM-OFDM

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Abstract—Adaptive coding and modulation is an important topic considering future communication systems. Orthogonal Frequency Division Multiplexing (OFDM) has been identified as a promising technique, which offers the possibility for further enhancements by bit and power loading schemes. Commonly, channel coding has not been considered in the optimization of such algorithms. It is, however, an important component used in nearly every communication system. In this paper we propose a new scheme to adapt code rate, modulation and transmit power by solving a convex optimization problem based on a bisection approach in order to enhance the frame error rate at a fixed target rate.

I. INTRODUCTION

The adaptation of communication systems to the current channel state is a crucial step towards higher spectral efficiencies and more robust systems. Obviously, the whole system has to be considered in the optimization of common parameters like modulation and power, which includes also the applied channel coding. This, however, has been neglected in previous works, e.g., [1]–[3]. Only recently some attention has been given to the consideration of channel coding in bit and power loading algorithms with respect to the capacity of bit interleaved coded modulation systems (BICM), e.g., [4]–[6]. Still, the information theoretical measure capacity does not describe the performance of coded systems completely and only holds for perfect capacity achieving codes. To this end, we propose an efficient extension to the approach of Krongold et al. [1], which uses the bisection method to solve the resulting convex optimization problem. Instead of analytical error rate expressions the simulated AWGN performance of a set of code and modulation combinations (modes) will be used to form a look-up table describing the required signal-to-noise-ratio (SNR) for a specific subcarrier rate. The codes will be chosen from a common code family, e.g., convolutional codes with a fixed constraint length, realizing different code rates. Adapting the code rate additionally allows for higher flexibility in comparison to fixed code rate scenarios. A similar approach has been proposed by Stiglmayr et al. [7], however, solving a rate optimization problem formulated in terms of the BICM capacity by linearization neglecting a finer grained power control.

The remainder of this paper is organized as follows. The system model used throughout the paper is introduced in

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Section II, in Section III the performance measure, which is used in Section IV to obtain an optimization algorithm, is characterized. In Section V performance results for several codes are shown and compared to other known loading approaches. Finally, in Section VI this paper is concluded.

Notation

In the following, vectors and sets are denoted by lower case bold and calligraphic letters, respectively. Furthermore, probabilities are denoted as P . $\mathcal{N}_C(\mu, \sigma^2)$ describes a complex Gaussian distribution with mean μ and variance σ^2 .

II. SYSTEM MODEL

We consider an equivalent baseband model of an OFDM system with N_C subcarriers assuming perfect synchronisation, a sufficient guard interval (GI) and perfect knowledge of the channel state information (CSI) at both transmitter and receiver. Thus, the system can be described in frequency domain as

$$y_k = h_k \cdot \sqrt{p_k} \cdot d_k + n_k, \quad (1)$$

where h_k denotes the channel coefficient in frequency domain on subcarrier $k = 1, \dots, N_C$ and p_k , d_k , n_k and y_k denote the transmit power, the transmit symbol, the Gaussian noise and the receive signal, respectively. The overall transmit power is given by $\mathcal{P} = \sum_{k=1}^{N_C} p_k$ and the power of the noise $n_k \sim \mathcal{N}_C(0, \sigma_n^2)$ is fixed to $\sigma_n^2 = 1$. The N_C frequency domain channel coefficients are determined by

$$h_k = \sum_{\ell=0}^{L_F-1} \tilde{h}(\ell) e^{-j\Omega_k \ell}, \quad (2)$$

where the L_F taps of the time domain channel are $\tilde{h}(\ell) \sim \mathcal{N}_C(0, 1/L_F)$ and $\Omega_k = 2\pi/N_C(k-1)$ denotes the k -th normalized equidistant sampling frequency.

A. Modulation

Throughout this paper transmit symbols stemming from M -QAM (\sqrt{M} -ASK) modulation alphabets \mathcal{A} with binary reflected gray mapping are considered. To each subcarrier k an individual alphabet of cardinality $M_k = |\mathcal{A}_k|$ may be assigned. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder.

As any square M -QAM can be represented by two \sqrt{M} -ASK without loss, we will constrain the following descriptions

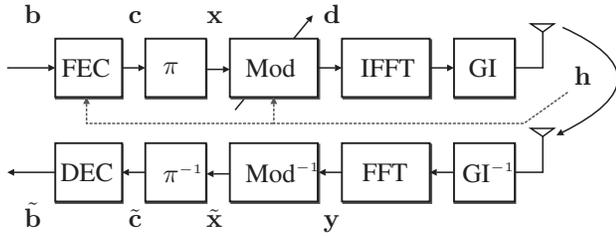


Fig. 1. System Model of an adaptive BICM-OFDM system.

to ASK constellations. An expansion to QAM constellations can easily be obtained by simply halving the power and rate constraint while doubling the resulting powers and rates.

B. Coding

Fig. 1 shows the general system model including channel coding and interleaving. The applied coding scheme uses a single forward error correction (FEC), which encodes the information bits of one OFDM symbol. Non-systematic non-recursive convolutional encoders of rates $R_C \in \{1/4, 1/3, 1/2, 2/3, 3/4\}$ with constraint lengths $L_C \in \{3, 7\}$ are considered.

In all cases, the code word length is fixed to the number of bits in one OFDM symbol, leading to longer code words for higher data rates. Thus, no time diversity is exploited. A BCJR algorithm is used for soft-decoding and random interleaving is applied.

III. CODED SYSTEM PERFORMANCE

The bit error rate performance of uncoded \sqrt{M} -ASK transmission is a well known property, which can be described as a function of the signal-to-noise ratio γ (SNR) by

$$P_{b,\sqrt{M}\text{-ASK}} = \frac{2}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc} \left(\sqrt{\frac{3}{M-1}} \gamma \right). \quad (3)$$

Accordingly, the frame error rate (FER) given a certain frame length L_N - the number of channel uses - is defined as

$$P_f = 1 - (1 - P_{b,\sqrt{M}\text{-ASK}})^{L_N \log_2 \sqrt{M}}, \quad (4)$$

which can be used to derive the SNR γ required to achieve a given bit or frame error rate for a certain modulation. This is the basis for many known bit loading algorithms, e.g., [1]. However, these results are limited to uncoded systems. To capture the behavior of the whole system - including an appropriate channel code - (3) and (4) are not sufficient. One way to obtain a measure of quality is the simulated performance of the coded system.

In order to characterize the SNR, which is necessary on a subcarrier to achieve a given frame error rate performance, simulations of a system with equivalent block length $L_N = N_C$ and AWGN noise are sufficient. The reasoning behind this is, that given the SNR of a single subcarrier γ_k we assume all symbols of the code word to have SNR γ_k . The error rate performance of a specific code and modulation disturbed

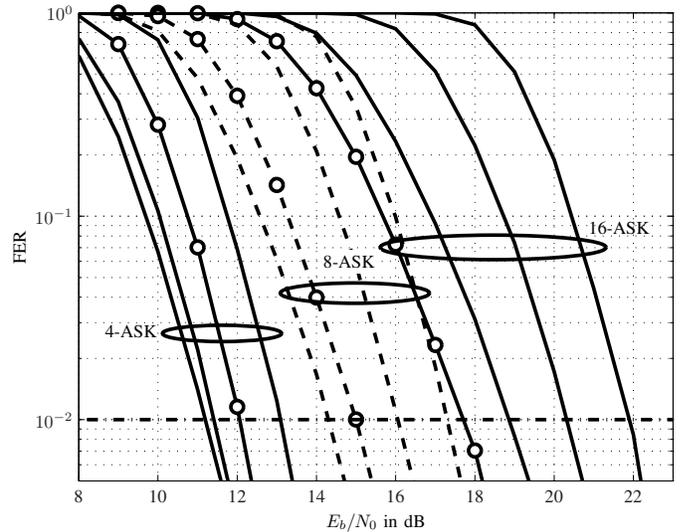


Fig. 2. FER vs. E_b/N_0 comparison of some combinations of \sqrt{M} -ASK with $\log_2(\sqrt{M}) = 2, 3, 4$ and convolutional codes $R_C \in \{1/4, 1/3, 1/2, 2/3\}$ (left to right) with $L_C = 3$. Circles denote modes with 1 bit/s/Hz. Frame length has been chosen $L_N = 1024$.

by AWGN at γ_k indicates, which mode may be chosen to achieve some frame error rate constraint. Using this heuristic, the overall error rate of an OFDM symbol with different SNRs and properly chosen subcarrier modes can be assumed to fulfill the error rate constraint.

Still, the AWGN assumption is optimistic in the sense, that individual channel properties have been compensated properly, but offers a good indication of the necessary SNR at a specific data rate to support a target frame error rate on a single subcarrier. Even though the applied channel code could cope with SNR differences over subcarriers an ergodic Rayleigh fading channel would lead to far too pessimistic performance measures because of subcarriers with very low SNRs. Such subcarriers will be compensated for in a perfect adaptive system by the assignment of more power, a different modulation and stronger coding.

Fig. 2 shows the FER results of Monte-Carlo simulations for \sqrt{M} -ASK constellations up to $\sqrt{M} = 2^4$ and a variety of code rates versus E_b/N_0 . It is quite clear, that only a subset of combinations will actually be used due to the fact, that at a fixed rate one code-modulation combination will lead to the best performance, e.g., 4-ASK with a half rate code vs. 16-ASK with a quarter rate code achieve the same spectral efficiency of 1 bit/s/Hz (circles) at a much lower E_b/N_0 . Based on these simulation results, the system performance can be characterized as will be shown in the next Section.

IV. POWER OPTIMIZATION

The problem statement we have to solve to enhance the error rate performance is the well known optimization problem (5). By this approach the transmit power \mathcal{P} is minimized given a target rate R_{Total} and frame error rate P_{Target} , where R_{Total} denotes the overall target rate and a local code rate $R_{C,k}$ together with the applied modulation $\sqrt{M_k}$ define the bit rate

$r_k = \log_2(\sqrt{M_k})R_{C,k}$ on subcarrier k . Note, that the optimum solution at one target FER scaled to the available transmit powers can be used to show performance gains in terms of the error rate.

$$\begin{aligned} & \text{minimize} && \mathcal{P} = \sum_k^{N_C} p_k \\ & \text{subject to} && \sum_k^{N_C} r_k = R_{\text{Total}} \quad \text{and} \quad P_f < P_{\text{Target}}. \end{aligned} \quad (5)$$

The FER constraint P_{Target} leads to a description of the interdependence of rate and power. The results in Section III at the target FER lead to rate-power pairs defining potential subcarriers modes. Accordingly, Fig. 3 shows all rate-power points up to a maximum rate $R_{\text{max}} = 4$ at a FER $P_{\text{Target}} = 10^{-2}$ found by Monte-Carlo simulations of the AWGN performance of an equivalent system with frame length N_C and the analytical frame error rate expression for uncoded ASK constellations (4). In a perfect adaptive system subcarrier channel differences would be exploited or compensated by assignment of power, modulation alphabet and code rate, making the AWGN performance a good quality measure to identify the SNR requirement $\hat{\gamma}_i$ of a mode i .

The set of all rate-power points is therefore defined as $\mathcal{S} = \{(R_i, \hat{\gamma}_i) | f_{M_i, R_{C,i}}(p_i) = P_{\text{target}}\}$, where $f_{M_i, R_{C,i}}$ denotes the frame error rate function, e.g., shown in Fig. 2, parametrized by the transmission parameters and the power. The SNR $\hat{\gamma}_i$ for the real valued system is defined as

$$\hat{\gamma}_i = R_{C,i} \log_2(\sqrt{M_i}) \frac{E_{b,i}}{N_0/2}. \quad (6)$$

where i is some index over all elements of the set \mathcal{S} .

To solve (5) efficiently, convexity has to be ensured. To this end, a convex set of rate-power points $\mathcal{C} \subset \mathcal{S}$ has to be found, which will be discussed in Section IV-A. Based on this, the Lagrangian of (5) can be derived, allowing for an efficient solution by the well known bisection approach discussed in Section IV-B.

A. Convexity

As previously mentioned a convex set \mathcal{C} is required, which can be constructed by the convex hull of the whole set as shown by the solid line in Fig. 3, where [8] has been used. The resulting look-up table for rate R_i , SNR requirement $\hat{\gamma}_i$, code rate $R_{C,i}$ and modulation size $\sqrt{M_i}$, where i is simply some index over all elements of \mathcal{C} , is shown in Table I. All combinations of coding (or no coding) and modulation resulting in a rate smaller than $R_{\text{max}} = 4$ have been considered. Note, that the relatively weak convolutional code is only advantageous at lower rates, e.g., a combination of 16-ASK with code of rate $R_C = 1/2$ needs much more power, i.e., approx. 20.5dB, at a given error rate of $P_{\text{Target}} = 10^{-2}$ than simply using uncoded 4-ASK with approx. 17dB. Such a table has to be generated once for a set of system parameters, i.e., N_C , code ensemble and maximum allowed rate R_{max} .

Based on such an easily storable look-up table the rate of each OFDM symbol can be optimized. More specifically, the

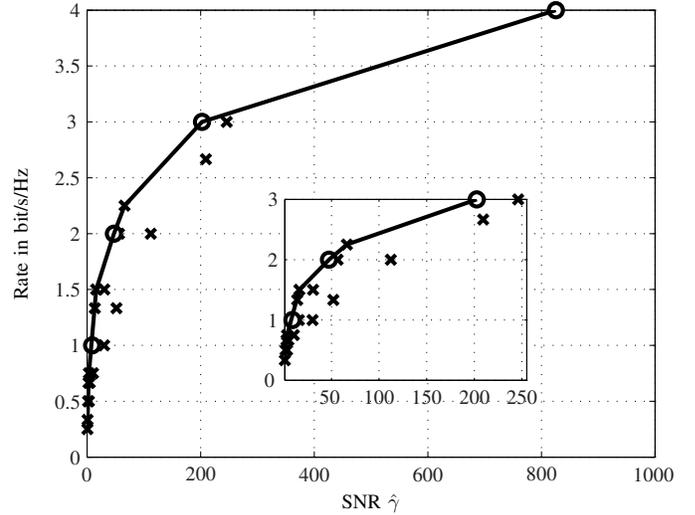


Fig. 3. Set of all rate-power points (x - coded modes; o - uncoded modes) and its convex hull (solid line) at $P_{\text{Target}} = 10^{-2}$; $L_N = 1024$ and $L_C = 3$.

TABLE I
LOOK-UP TABLE FOR $L_N = 1024$, $L_C = 3$ AND TARGET FER
 $P_{\text{TARGET}} = 10^{-2}$

Rate	$\hat{\gamma}$	R_C	$\log_2 \sqrt{M}$
0.25	0.88	0.25	1
0.33	1.17	0.33	1
0.50	1.84	0.50	1
0.67	2.83	0.67	1
0.75	3.40	0.75	1
1.50	16.36	0.75	2
2.00	47.49	1.00	2
2.25	66.22	0.75	3
3.00	202.57	1.00	3
4.00	825.51	1.00	4

look-up table has to return the mode with the greatest rate still being feasible. Feasibility in the bisection approach, which will be discussed shortly, is connected to the slope of the rate-power curve, given by $\delta \mathbf{R} / \delta \hat{\Gamma}$ with \mathbf{R} the vector of all rates on the convex hull and $\hat{\Gamma}$ the vector of the respective SNRs $\hat{\gamma}$.

B. Coded Bisection Approach

The stated optimization problem is well known and has been solved for the uncoded case by several approaches. One very efficient way to find the optimal solution given the Lagrangian formulation of the convex optimization problem (5) is the bisection approach, which has been applied to the uncoded bit and power loading problem by Krongold et. al [1]. The Lagrangian of (5) with respect to the equivalent minimization problem is

$$J(\lambda) = \sum_{k=1}^{N_C} p_k + \lambda \left(\sum_{k=1}^{N_C} r_k - R_{\text{Total}} \right). \quad (7)$$

Considering the unconstrained problem, i.e. neglecting R_{Total} , each λ corresponds to an optimal power and rate distribution, which minimizes the cost function $J(\lambda)$. Accordingly, if

Require: look-up table $L(\eta) \rightarrow r, p$
 $\mathcal{P}_{\text{low}} = 0, R_{\text{low}} = 0$
 $\mathcal{P}_{\text{high}} = \max(\Gamma) \sum_{k=1}^{N_C} 1/|h_k|^2, R_{\text{high}} = N_C R_{\text{max}}$

loop
 $\lambda = \frac{R_{\text{high}} - R_{\text{low}}}{\mathcal{P}_{\text{high}} - \mathcal{P}_{\text{low}}}$ {rate-power slope}
for $k = 1$ to N_C **do**
 $\eta = \lambda/|h_k|^2$ {modify with channel}
 $r_{\text{new},k} = L(\eta)$ {look-up best feasible rate}
 $p_{\text{new},k} = L(\eta)/|h_k|^2$ {get needed power}
end for
 $R_{\text{new}} = \sum_{k=1}^{N_C} r_{\text{new},k}$ {overall rate}
 $\mathcal{P}_{\text{new}} = \sum_{k=1}^{N_C} p_{\text{new},k}$ {overall power}
if $R_{\text{new}} == R_{\text{high}}$ or $R_{\text{new}} == R_{\text{low}}$ or $R_{\text{new}} == R_{\text{Target}}$ **then**
Set $\mathbf{p} = \mathbf{p}_{\text{new}}$ and $\mathbf{r} = \mathbf{r}_{\text{new}}$
END LOOP
else if $R_{\text{new}} < R_{\text{Total}}$ **then**
 $\mathcal{P}_{\text{low}} = \mathcal{P}_{\text{new}}, R_{\text{low}} = R_{\text{new}}$
else
 $\mathcal{P}_{\text{high}} = \mathcal{P}_{\text{new}}, R_{\text{high}} = R_{\text{new}}$
end if
end loop
return Rates \mathbf{r} and Powers \mathbf{p}

Fig. 4. Coded Bisection approach to rate optimization

$J(\lambda)/\delta r_k = 0, \forall k$, the condition

$$\frac{\delta p_k}{\delta r_k} = -\lambda \quad \forall k, \quad (8)$$

has to be fulfilled, meaning that the optimal rates and powers have to be chosen through that point on the rate-power curve with slope λ .

A look-up table, which provides the rate and power at a specific slope $\eta = \lambda/|h_k|^2$, has to be constructed from \mathcal{C} to calculate the optimal rates and powers on all subcarriers for a given λ . The optimal λ^* , which minimizes the power taking the target rate R_{Total} into account, can then be found iteratively.

The algorithm shown in Fig. 4 implements this procedure. Starting with the two extremes $\mathcal{P}_{\text{low}}, R_{\text{low}}$ (no power and rate) and $\mathcal{P}_{\text{high}}, R_{\text{high}}$ (maximum power and rate), optimal powers p_k and rates r_k are calculated via the look-up table. Following a bisection approach, the new overall power and rate are then used to calculate a new slope λ to the rate-power curve, solving the rate and power distribution again until the optimal solution is obtained.

The vectors $\mathbf{p} = [p_1, \dots, p_{N_C}]^T$ and $\mathbf{r} = [r_1, \dots, r_{N_C}]^T$ in Fig. 4 collect all subcarrier powers and rates, respectively. For each subcarrier k a power requirement $p_{\text{new},k}$ can be determined by look-up of the modified slope $\eta = \lambda/|h_k|^2$. The overall slope λ is then adjusted in the next iteration by the newly calculated power and rate boundaries testing these hypotheses until the overall power constraint is fulfilled (for more details see [1]). The resulting rate vector \mathbf{r} directly defines modulation and code rate per Table I.

The look-up table $L(\eta)$ is constructed such that the rate-power pair $(r_i, p_i) \in \mathcal{C}$ with the greatest rate r_i is chosen,

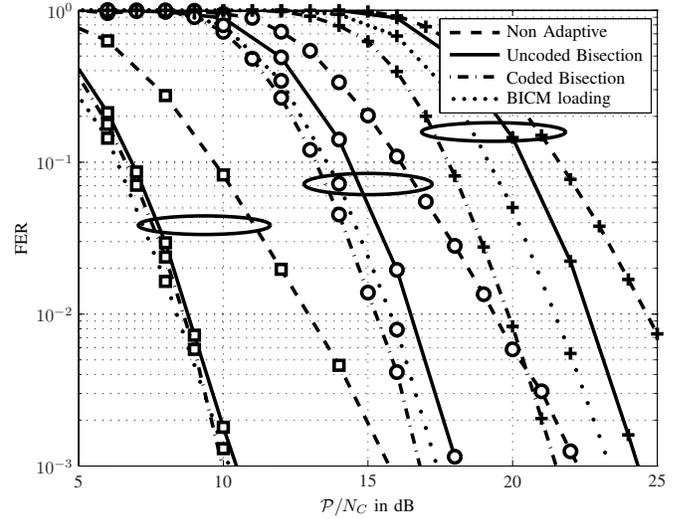


Fig. 5. FER vs. Average subcarrier SNR for 1,2 and 3 bit/s/Hz (squares, circles, crosses) for $N_C = 1024, L_C = 3$ and $L_F = 10$; Optimization parameters are $\mathcal{P}_{\text{Target}} = 10^{-2}$ and $R_{\text{max}} = 8$.

where

$$\eta < \left. \frac{\delta \mathbf{R}}{\delta \Gamma} \right|_{r_i} \quad (9)$$

is still fulfilled.

C. Code constraint

This bisection approach efficiently solves the convex optimization problem of the coded system by the previously explained steps. However, the optimization results in local subcarrier code rates, which is not a common solution in today's wireless systems and beyond the scope of this paper. Instead, a mean code rate is calculated to choose the code with the largest available code rate \bar{R}_C fulfilling $\bar{R}_C < \frac{1}{N_C} \sum_{k \in \mathcal{T}} R_{C,k}$ as the outer code for one OFDM symbol, where \mathcal{T} denotes the set of all nonzero local code rates $R_{C,k}$. Due to this solution, though, the target rate will no longer be achieved. As the subcarrier rates are changed by the application of a fixed global code rate, an overall rate loss is introduced (stronger error protection than required) violating the target rate constraint. A solution to this problem is a two step process, fixing the code rate in the first step followed by the optimization over a convex set of all modes given this code rate.

V. RESULTS

As described in the previous section, the results were obtained by optimizing the coded bit and power loading twice. The first run with variable coding to obtain a mean code rate and the second one to optimize the bit and power loading under this fixed code rate. By this approach, the rate requirement will be fulfilled at a previously chosen code rate \bar{R}_C . For simulations a fully complex valued system model was applied.

Fig. 5 shows results for a system with $N_C = 1024$ subcarriers at different target rates, using convolutional codes of constraint length $L_C = 3$. Optimization parameters have

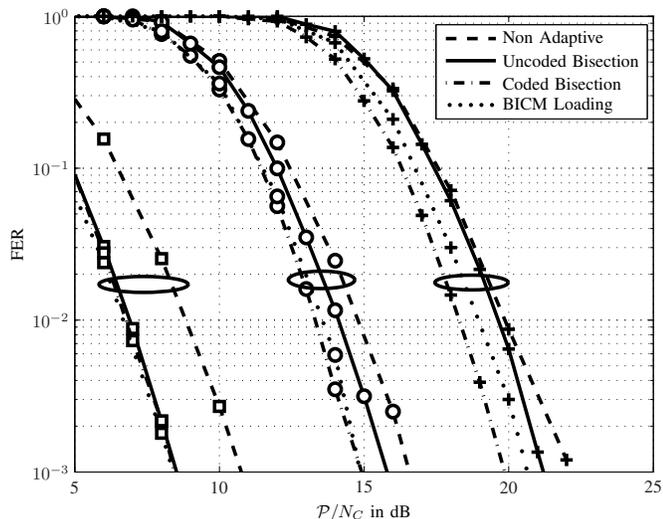


Fig. 6. FER vs. Average subcarrier SNR for 1, 2 and 3 bit/s/Hz (squares, circles, crosses) for $N_C = 1024$, $L_C = 7$ and $L_F = 10$; Optimization parameters are $P_{\text{Target}} = 10^{-2}$ and $R_{\text{max}} = 8$.

been chosen as $P_{\text{Target}} = 10^{-2}$ using a look-up table with a maximum rate $R_{\text{max}} = 8$ for the coded bisection approach. The uncoded bisection results were achieved with a target bit error rate of $P_{\text{Target}} = 10^{-3}$ and a maximum alphabet size $M_{\text{max}} = 10$ (1024-QAM). The chosen target FER P_{Target} is used in the optimization of all SNRs, showing the potential gains in terms of FER. The resulting optimum power allocation is scaled to the current transmit power level per SNR. As a further reference, results from [4] were added as "BICM loading", where bit and power loading based on the average mutual information of a code word and also adaptation of the interleaver are applied.

Note, that at lower spectral efficiencies the performance of the coded bisection is only slightly better than the uncoded bisection approach, which is due to the applied two step optimization. The optimal rate and power allocation only depends on the current channel realization and the chosen spectral efficiency. For 1 bit/s/Hz the mean code rate will either be $\bar{R}_C = 1/2$ or $\bar{R}_C = 2/3$ with high probability, which leads to nearly the same results as the uncoded bisection solutions with an outer half-rate code. In the high SNR regime, though, the gain is slightly increasing. A system applying adjustable local code rates would perform better in such cases. With increasing transmit rate the performance gain compared to the uncoded bisection solution increases as well. The chosen code rates will deviate from the half-rate code of the uncoded bisection towards higher code rates. Especially at higher rates significant gains can be achieved due to the adapted code rate.

In comparison to "BICM loading", which achieves better results than the uncoded bisection solution, the same observations hold true. At lower rates, both bisection methods are outperformed due to the interleaver adaptation, but with increasing spectral efficiency adaptation of the code rate becomes more and more important.

Fig. 6 shows results using a similar setup as before, but with convolutional codes of constraint length $L_C = 7$. The overall stronger codes limit the potential gains, as can be seen by direct comparison to Fig. 5. A longer constraint length leads to a more robust coding scheme, which is less affected by variations within one code word. Still, the uncoded bisection approach hardly achieves gains indicating the importance of a proper code rate, which is again confirmed by the "BICM loading" results. Similar observations can be found in many link adaptation schemes applying outer codes with a fixed code rate. With increasing spectral efficiency gains by pure bit and power loading schemes tend to zero, which has been analyzed with respect to the cdf of the bit level capacities in [9]. Obviously, the choice of a proper code rate has to be included in the overall optimization.

VI. CONCLUSION

The consideration of channel coding in the optimization of communication systems is crucial to enhance the performance and exploit the capabilities provided by channel coding. For coded modulation system we have detailed an extension of the bisection approach used in [1] in order to efficiently enhance the overall error rate performance. Due to the choice of a single outer code, it is important to pre-choose a code rate adapted to the channel state information and optimize the bit and power loading based on this. Unfortunately, this limits the potential gains as most subcarriers will not be used at their optimal rate. Future work should include the application of codes of different code rates to achieve a finer grained control. Nonetheless, our results show, that the bisection method using the simulated AWGN performance of different modes is a valid method to enhance the performance of communication systems.

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