RATE ENHANCEMENT OF BICM-OFDM WITH ADAPTIVE CODING AND MODULATION VIA A BISECTION APPROACH

Carsten Bockelmann, Dirk Wübben and Karl-Dirk Kammeyer

University of Bremen, Germany
Department of Communications Engineering
Email: {bockelmann, wuebben, kammeyer}@ant.uni-bremen.de

ABSTRACT

Adaptive communication is an important topic to enhance the capabilities of future communication systems. Especially in combination with Orthogonal Frequency Division Multiplexing (ODFM) bit and power loading schemes have been devised to enhance the transmit rate of communication systems. The consideration of channel coding in the optimization is an obvious requirement. In the past, though, mainly uncoded systems have been investigated allowing for further enhancements. In this paper we propose a new scheme to adapt code rate and modulation by solving the convex optimization problem based on a bisection approach in order to maximize the achievable data rate at a fixed target error rate.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing is a key technology to efficiently overcome frequency selective channels. Especially in wireline communications, but also in several wireless standards, OFDM has been widely applied. Based on the orthogonality of the subcarriers of OFDM systems several approaches to adapt communication to the current channel conditions have been proposed. However, these so called bit and power loading algorithms mostly focus on the uncoded bit error rate (BER), which can be described analytically for QAM and PSK constellations. Adaptive systems in general have been studied extensively in the past years, but the aspect of practical channel coding is still often neglected for the sake of analytical or purely information theoretical solutions. Several efficient solutions have been proposed to the uncoded optimization problem, e.g. [1, 2], but only recently the regard of channel coding has become a topic.

The specific problem of coded bit and power loading for single antenna OFDM systems has been addressed by Li et al. [3], proposing a solution motivated by bit interleaved coded modulation (BICM) [4] capacity results. Similarly, Sankar et al. [5] proposed a scheme based on a simple approximation of the BICM capacity and derived an appropriate waterfilling solution to that problem. Another approach quite akin to [3], but based on SNR thresholds, has been proposed by Stiglmayr et al. [6] to optimize the performance of an OFDM system, where loading is only applied to groups of subcarriers called chunks. For MIMO system the authors have proposed an adaptive algorithm based on an approximation of the coded modulation capacity regarding both error rate and transmit rate optimization [7].

In this paper we propose a new scheme, which expands the bisection approach originally used by Krongold et al. [2] to adapt power, modulation and the code rate of a fixed channel code (e.g., convolutional or turbo code) based on a set of rate-power points, which are derived from AWGN simulation results of the applied code. This efficient solution is compared to other uncoded and coded loading approaches and is shown to achieve superior performance in comparison to other approaches.

The remainder of this paper is organized as follows. The system model used throughout the paper is introduced in Section 2, in Section 3 the system performance, which is used in Section 4 to obtain an optimization algorithm is characterized. In Section 5 performance results for several codes are shown and compared to other known loading approaches. Finally, in Section 6 this paper is concluded.

Notation

In the following, vectors are denoted by lower case bold letters. Furthermore, probabilities are denoted as P. $\mathcal{N}_C(\mu, \sigma^2)$ describes a complex Gaussian distribution with mean μ and variance σ^2.

2. SYSTEM MODEL

We consider an equivalent baseband model of an OFDM system with N_C subcarriers with perfect knowledge of the channel state information at both transmitter and receiver. Thus, the system model in frequency domain is

$$y_k = h_k \cdot \sqrt{p_k} \cdot d_k + n_k \,, \qquad (1)$$

where h_k denotes the channel coefficient in frequency domain on subcarrier $k = 1, \cdots, N_C$ and p_k, d_k, n_k and y_k

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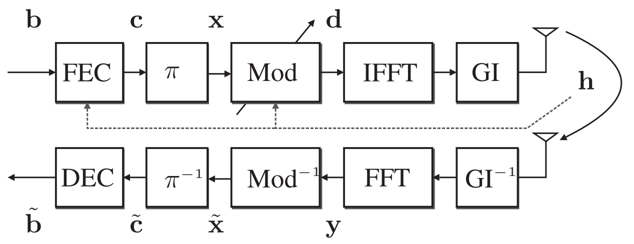

Fig. 1. System Model

denote the transmit power, the transmit symbol, the Gaussian noise and the receive signal, respectively. The overall transmit power is given by $\mathcal{P} = \sum_{k=1}^{N_C} p_k$ and the power of the noise $n_k \sim \mathcal{N}_C(0, \sigma_n^2)$ is fixed to $\sigma_n^2 = 1$. The N_C frequency domain channel coefficients are determined by $h_k = \sum_{\ell=0}^{L_F-1} \tilde{h}(\ell) e^{-j\Omega_k \ell}$, where the L_F taps of the time domain channel are $\tilde{h}(\ell) \sim \mathcal{N}_C(0, 1/L_F)$ and $\Omega_k = 2\pi/N_C\,(k-1)$ denotes the k-th normalized equidistant sampling frequency.

2.1. Modulation

Throughout this paper transmit symbols stemming from M-QAM ($\sqrt{M}$-ASK) modulation alphabets $\mathcal{A}$ with binary reflected gray mapping [8] are considered . To each subcarrier k an individual alphabet of cardinality $M_k = |\mathcal{A}_k|$ may be assigned. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder.

As any square M-QAM can be represented by two $\sqrt{M}$-ASK without loss, we will constrain the following descriptions to ASK constellations. An expansion to QAM constellations can easily be obtained by simply halving the power constraint and maximum rate while doubling the resulting powers and rates.

2.2. Coding

Fig. 1 shows the general system model including the channel code and interleaving. Two codes are applied, i.e., non-systematic non-recursive convolutional encoders of rates $R_C \in \{1/4, 1/3, 1/2, 2/3, 3/4\}$ and constraint lengths $L_C \in \{3, 7\}$.

In all cases, the code word length is fixed to the number of bits in one OFDM symbol, leading to longer code words for higher data rates. Thus, no time diversity is exploited. A BCJR algorithm has been used for soft-decoding and the interleaver was designed randomly.

3. CODED PERFORMANCE

The bit error rate performance of $\sqrt{M}$-ASK neglecting any channel code is a well known property, which can be described in dependence of the signal-to-noise ratio γ (SNR) by

$$\mathrm{P}_b = \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{3}{M-1}\gamma}\right). \quad (2)$$

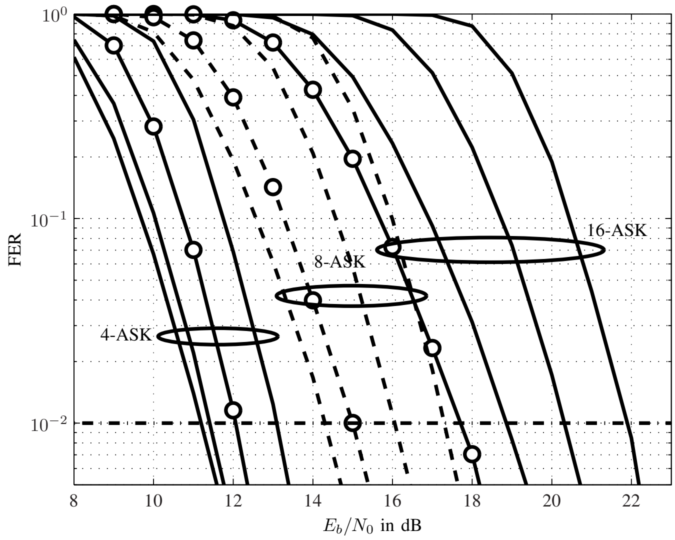

Fig. 2. FER vs. E_b/N_0 comparison of some combinations of $\sqrt{M}$-ASK with $\log_2(\sqrt{M}) = 2, 3, 4$ and convolutional codes $R_C \in \{1/4, 1/3, 1/2, 2/3\}$ (left to right) with $L_C = 3$ and frame length $L_N = 1024$.

Accordingly, the frame error rate (FER) is obtainable given a certain frame length L_N (number of channel uses) as

$$\mathrm{P}_f = 1 - (1 - \mathrm{P}_b)^{L_N \log_2 \sqrt{M}}, \quad (3)$$

which can be used to derive the SNR $\hat{\gamma}$ required to achieve a given bit P_b or frame error rate P_f. This is the basis for many known bitloading algorithms, e.g., [2]. However, these results are limited to uncoded systems. To capture the behavior of the whole system - including an appropriate channel code - (2) and (3) are not sufficient. One way to obtain a measure of quality is the simulated performance of the system.

In order to characterize the SNR γ, which is necessary to achieve a given error rate performance P_f, simulation of a system with equivalent block length $L_N = N_C$ and an AWGN channel is sufficient. The effective subcarrier SNR given by some assigned power p_k and the channel power $|h|^2$ can then be compared to these thresholds to identify suitable modes. This AWGN assumption is optimistic in the sense, that individual channel properties (Rayleigh fading, correlation) have been compensated properly, but offers a good indication which SNR is needed at a specific data rate on a single subcarrier to support a target frame error rate. Ergodic Rayleigh fading would lead to far too pessimistic thresholds because of subcarriers with very low SNRs. Such subcarriers will be compensated for in a perfect adaptive system by the assignment of more power, a different modulation and stronger coding.

Fig. 2 shows the FER results of Monte-Carlo simulations for $\sqrt{M}$-ASK constellations up to $\sqrt{M} = 2^4$ and a variety of code rates versus E_b/N_0. It is quite clear, that only a subset of combinations will actually be used due to the fact, that at a fixed rate one code-modulation combination will lead to the best performance, e.g., 4-ASK with a half rate code vs. 16-ASK with a quarter rate code achieves the same performance at a much lower E_b/N_0 at rate 1 bit/s/Hz (circles).

Based on these simulation results, the system performance can be characterized as will be shown in Section 4.

4. RATE OPTIMIZATION

To enhance the data rate per OFDM symbol for the system defined in Section 2 we have to solve the well known optimization problem

$$\text{maximize} \quad R_{\text{Total}} = \sum_{k}^{N_C} r_k$$

$$\text{subject to} \quad \sum_{k}^{N_C} p_k = P \quad \text{and} \quad \mathrm{P}_f < \mathrm{P}_{\text{Target}}\,, \qquad (4)$$

where $r_k = \log_2(\sqrt{M_k})R_{C,k}$ is the rate of subcarrier k given by a local code rate $R_{C,k}$ and modulation alphabet of cardinality $\sqrt{M_k}$ and p_k is the transmit power. The frame error rate constraint $\mathrm{P}_{\text{Target}}$ ensures a certain quality of service, but is also a requirement to relate the powers p_k to the achievable rates r_k. From the results of Section 3 we can conclude, that at a target error rate $\mathrm{P}_{\text{target}}$ each combination of code and modulation leads to a specific power requirement to achieve the corresponding rate. Fig. 3 shows all possible rate-power pairs at a FER of $\mathrm{P}_{\text{Target}} = 10^{-2}$ with maximum rate of $R_{\max} = 4$. Included are all points found by Fig. 2 and uncoded modes following (3) to consider higher SNRs, where coding may not be necessary any more. This gives the set of all rate-power points $\mathcal{S} = \{(R_i, \hat{\gamma}_i) | f_{\sqrt{M_i}, R_{C,i}}(p_i) = \mathrm{P}_{\text{target}}\}$, where $f_{M_i,R_{C,i}}$ denotes the bit or frame error rate function, e.g., shown in Fig. 2, parametrized by the transmission parameters and the power. The SNR $\hat{\gamma}$ for the real valued system is defined as $\hat{\gamma}_k = R_{C,k}\log_2(\sqrt{M_k})\frac{E_{b,k}}{N_0/2}$.

4.1. Convexity

In order to solve problem (4) efficiently via the bisection method, the set $\mathcal{S}$ has to be convex. This whole set, though, is not a convex one. However, standard methods to identify the convex hull of a set of points can be used to build a convex subset $\mathcal{Q} \subset \mathcal{S}$, e.g. [9], which is indicated by the solid line in Fig. 3.

The resulting convex set of rate-power points is shown in Table 1. Based on such an easily storable look-up table, which has to be created once for a specific frame length and set of modes, the rate of an OFDM symbol can be optimized. More specifically, the look-up table has to return the mode with the greatest rate still being feasible. Feasibility in the bisection approach is connected to the slope of the rate-power curve, which can be calculated as $\delta\mathbf{R}/\delta\hat{\mathbf{\Gamma}}$ with $\mathbf{R}$ the vector of all rates on the convex hull and $\hat{\mathbf{\Gamma}}$ the vector of the respective SNRs.

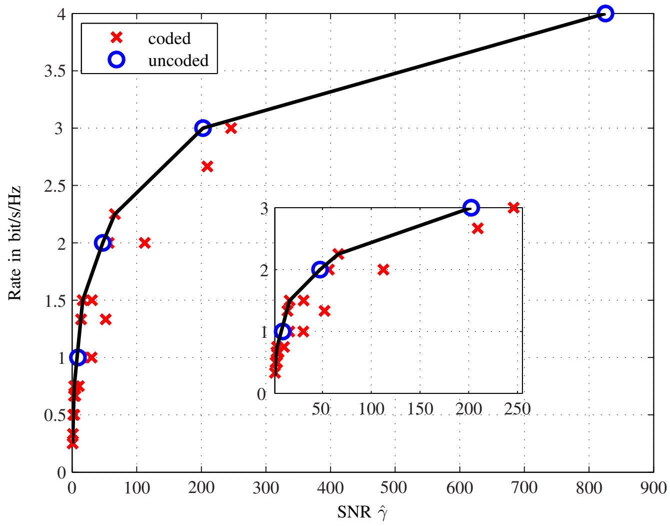

Fig. 3. Rate-power pairs at a FER of 10^{-2}, the set $\mathcal{S}$ is shown by 'x' (convolutional code $L_C = 3$), 'o' (uncoded) and the convex subset $\mathcal{Q}$ is visualized by the solid line

Table 1. Look-up table for $L_N = 1024$, $L_C = 3$ and $\mathrm{P}_{\text{Target}} = 10^{-2}$

Rate	$\hat{\gamma}$	R_C	$\log_2\sqrt{M}$
0.25	0.88	0.25	1
0.33	1.17	0.33	1
0.50	1.84	0.50	1
0.67	2.83	0.67	1
0.75	3.40	0.75	1
1.50	16.36	0.75	2
2.00	47.49	1.00	2
2.25	66.22	0.75	3
3.00	202.57	1.00	3
4.00	825.51	1.00	4

4.2. Coded Bisection Approach

The stated optimization problem is well known and has been solved for the uncoded case by several approaches. One very efficient way to find the optimal solution given the Lagrangian formulation of the convex optimization problem (4) is the bisection approach, which has been applied to the uncoded bit and power loading problem by Krongold et. al [2]. The Lagrangian of (4) with respect to the equivalent minimization problem is

$$J(\lambda) = -\sum_{k=1}^{N_C} r_k + \lambda\left(\sum_{k=1}^{N_C} p_k - P\right). \qquad (5)$$

Require: look-up table $L(\eta) \rightarrow r, p$

$\mathcal{P}_{\text{low}} = 0$, $R_{\text{low}} = 0$

$\mathcal{P}_{\text{high}} = \max(\hat{\mathbf{\Gamma}}) \sum_{k=1}^{N_C} 1/|h_k|^2$, $R_{\text{high}} = N_C R_{\max}$

loop

$\lambda = \frac{R_{\text{high}} - R_{\text{low}}}{\mathcal{P}_{\text{high}} - \mathcal{P}_{\text{low}}}$ {rate-power slope}

for $k = 1$ to N_C **do**

$\eta = \lambda/|h_k|^2$ {modify with channel}

$r_{\text{new},k} = L(\eta)$ {look-up best feasible rate}

$p_{\text{new},k} = L(\eta)/|h_k|^2$ {get needed power}

end for

$R_{\text{new}} = \sum_{k=1}^{N_C} r_{\text{new},k}$ {overall Rate}

$\mathcal{P}_{\text{new}} = \sum_{k=1}^{N_C} p_{\text{new},k}$ {overall Power}

if $\mathcal{P}_{\text{new}} == \mathcal{P}_{\text{high}}$ or $\mathcal{P}_{\text{new}} == \mathcal{P}_{\text{low}}$ **then**

Set $\mathbf{p} = \mathbf{p}_{\text{low}}$ and $\mathbf{r} = \mathbf{r}_{\text{low}}$

END LOOP

else if $\mathcal{P}_{\text{new}} == \mathcal{P}$ **then**

Set $\mathbf{p} = \mathbf{p}_{\text{new}}$ and $\mathbf{r} = \mathbf{r}_{\text{new}}$

END LOOP

else if $\mathcal{P}_{\text{new}} < \mathcal{P}$ **then**

$\mathcal{P}_{\text{low}} = \mathcal{P}_{\text{new}}$, $R_{\text{low}} = R_{\text{new}}$

$\mathbf{r}_{\text{low}} = \mathbf{r}_{\text{new}}$, $\mathbf{p}_{\text{low}} = \mathbf{p}_{\text{new}}$

else

$\mathcal{P}_{\text{high}} = \mathcal{P}_{\text{new}}$, $R_{\text{high}} = R_{\text{new}}$

end if

end loop

return Rates $\mathbf{r}$ and Powers $\mathbf{p}$

Fig. 4. Coded Bisection approach to rate optimization

The optimum value λ^* maximizing the rate given the total power $\mathcal{P}$ can then be found via the algorithm shown in Fig. 4, where $\mathbf{p} = [p_1, \ldots, p_{N_C}]^T$ denotes the vector of all subcarrier powers. The variable λ can be interpreted as the slope of the rate-power curve at the optimal point leading to a certain total power and rate. Starting with the two extremes $\mathcal{P}_{\text{low}}, R_{\text{low}}$ (no power and rate) and $\mathcal{P}_{\text{high}}, R_{\text{low}}$ (maximum power and rate) the bisection approach is used to calculate a new slope λ to the rate-power curve. Based on this, for each subcarrier the slope is modified by the power of the channel coefficient to find the supported rates. The slope λ is then adjusted in each iteration by the newly calculated power and rate boundaries testing these hypotheses until the overall power constraint is fulfilled (for more details see [2]).

The look-up table $L(\eta)$ is constructed such that the greatest rate r_i is chosen, where $\eta < \delta\mathbf{R}/\delta\hat{\mathbf{\Gamma}}|_{r_i}$ is still fulfilled. Furthermore, the vector of applied rates $\mathbf{r}$ directly defines modulation and code rate (e.g. Table 1). However, the application a specific code rate is beyond the scope of this paper. Therefore, an average code rate $\bar{R}_C = \frac{1}{N_C}\sum_{k=1}^{N_C} R_{C,k}$ is used to identify the best suited code, where the greatest available code rate smaller than $\bar{R}_C$ is applied leading to a slight loss in performance.

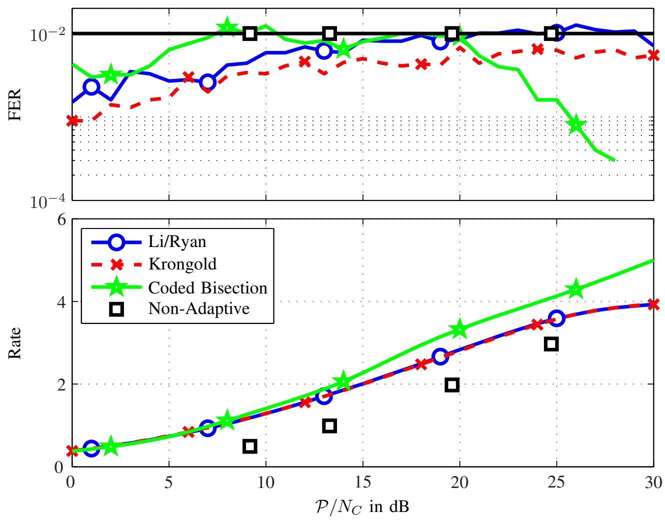

Fig. 5. FER and Rate vs average power per subcarrier $\mathcal{P}/N_C$ for convolutional codes of constraint length $L_C = 3$; $N_C = 1024$ subcarriers, $L_F = 10$ channel taps, target FER of $\mathrm{P}_{\text{Target}} = 10^{-2}$

5. RESULTS

Assuming that erroneous frames would be retransmitted in a communication system applying some form of automatic repeat request (ARQ), the following rate results are calculated considering only error free frames. For the sake of simplicity, this approach does not consider any form of hybrid ARQ schemes, were transmitted bits of an erroneous frame would not simply be thrown away.

Fig. 5 shows the FER and rate results versus the average subcarrier SNR for a convolutionally coded OFDM system with $N_C = 1024$ subcarriers, constraint length $L_C = 3$, $L_F = 10$ channel taps and a target frame error rate of $\mathrm{P}_{\text{Target}} = 10^{-2}$ with the maximum rate $R_{\max} = 4$ bit/s/Hz per real dimension. Due to the choice of a fixed code rate over one OFDM symbol and a maximum modulation of 16-ASK per dim., however, the maximum overall rate may be $R_{\max,\text{ov}} = 2 \cdot 2 \cdot 3/4 = 6$ (16-ASK per dim. with rate 3/4 code). Besides the presented Coded Bisection approach two additional adaptive algorithms have been depicted. The original uncoded bisection solution from [2] with a target bit error rate of $\mathrm{P}_{b,\text{Target}} = 0.02$ and Algorithm 3 from [3] (in the following "Li/Ryan") with a mutual information threshold of $\mathrm{MI} = 0.92$, which corresponds to the target FER. Both use a code of rate $R_C = 1/2$ with a maximum modulation of 16-ASK, hence an overall maximum rate of 4 bits/s/Hz is achievable. Furthermore, the non-adaptive result is depicted as a baseline to show the achieved gains of coded bit and power loading.

Clearly, the adaptation of code rate and modulation offers higher flexibility to cope with a given channel situation than

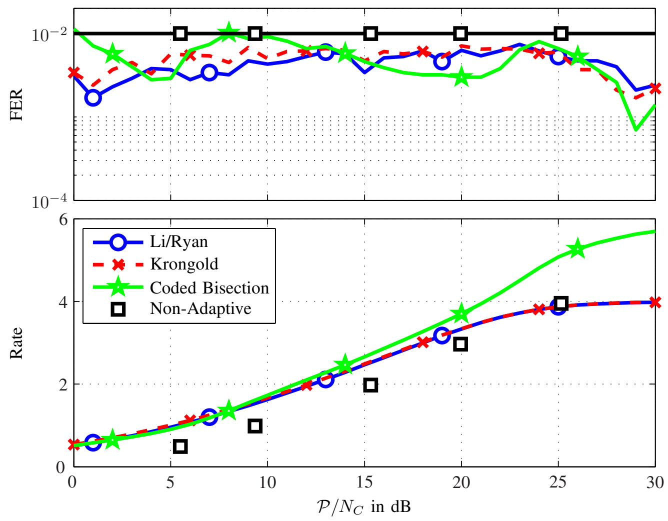

Fig. 6. FER and Rate vs average power per subcarrier $\mathcal{P}/N_C$ for convolutional codes of constraint length $L_C = 7$; $N_C = 1024$ subcarriers, $L_F = 10$ channel taps, target FER of $\mathrm{P}_{\mathrm{Target}} = 10^{-2}$

a fixed code rate scheme. Nonetheless, all adaptive schemes achieve significant rate gains w.r.t. rate. Note, that "Li/Ryan" offers no apparent enhancement in comparison to the uncoded bit and power loading. Furthermore, the target FER is maintained quite well with one exception. At high SNR, even though the maximum assignable rate is not yet achieved, the FER of our scheme decreases rapidly. This effect can be explained by the fixed frame code rate. Specifically, the code rate will be limited to the maximum of $R_C = 3/4$ even if the mean code rate is actually higher (e.g., nearly 1). Accordingly, the error protection capabilities exceeds the one resulting from the optimization lowering the transmit rate considerably.

Fig. 6 shows the results for the same system with convolutional codes of constraint length $L_C = 7$, resulting in better error protection. The original uncoded bisection solution uses a target bit error rate of $\mathrm{P}_{b,\mathrm{Target}} = 0.06$ and "Li/Ryan" a mutual information threshold of $\mathrm{MI} = 0.8$. Basically, the non-adaptive throughput is higher compared to $L_C = 3$ due to the stronger error protection capability, which allows for a higher rate at the same target FER. Overall gains due to adaptivity are slightly lessened because of the limited degrees of freedom the system offers. A stronger channel code makes better use of the existing frequency diversity, thereby limiting the possible gains. Especially at high SNR, though, the stronger error protection leads to different results than for $L_C = 3$, actually coping much better with the code rate choice. Quite obviously the overall code rates will be lower at the same SNRs in comparison to $L_C = 3$, because the look-up table will contain less uncoded modes owing to the stronger error protection.

6. CONCLUSION

The proposed approach is a very efficient means to enhance the rate of BICM-OFDM systems as the required look-up table can be computed beforehand. In contrast to [3], which works well, but does not lead to higher gains in comparison to the uncoded bit loading solution [2], our scheme achieves considerable gains especially in the high SNR range. The limiting factor for fixed code rate solutions is the highest achievable data rate given by code rate and maximum modulation size. Using a variety of code rates and considering uncoded transmission modes offers a higher flexibility for adaptation. Especially in the case of weaker convolutional codes uncoded modulation is a feasible solution allowing for substantially higher rates in the optimization. Fixing the code rate for one code word, though, again limits the performance and produces some side effects for all codes at higher SNRs. A finer grained control over the code rate would help to compensate for that, but that may not always be possible due to the nature of the applied channel coding. Another straightforward solution is offered by a two step loading, fixing the code rate in the first step and optimizing the loading with regard to this code rate in the second step. The overall complexity is of course doubled, as the algorithm has to be run twice with different look-up tables. Nonetheless, it is still a very efficient means to adapt the transmission of practical systems with respect to the actual performance of the applied channel codes.

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