Minimum MSE Relaying for Arbitrary Signal Constellations in Coded Relay Networks

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***Abstract*—In this paper a soft relaying scheme for coded relay networks combining the benefits of classical Decode-Forward (DF) and Amplify-Forward (AF) is extended to higher order modulation schemes. In order to minimize the mean-squared-error at the destination, the conditional expectation value of the symbol after soft-output channel decoding at the relay is transmitted. The main idea of this scheme called Decode-Estimate-Forward (DEF) is to exploit coding gain like DF while still preserving reliability information as AF. This approach was presented by the authors for BPSK in coded systems, but the extension to arbitrary modulation alphabets presented here allows for more flexible system designs. The performance of the proposed relay function is compared to classical AF and DF in a wide variety of scenarios like AWGN and Rayleigh fading channels as well as different numbers and constellations of relays.**

I. Introduction

Soft information relaying has attracted increasing attention in the last years. This approach combines the advantages of the classical relay protocols Amplify-Forward (AF) and Decode-Forward (DF) [1]. DF makes use of the discrete alphabet and of the coding gain in a coded system, but suffers from error propagation in the case of decoder failure at the relay. AF ignores the benefits of channel coding and discrete alphabets, but avoids error propagation and preserves reliability information. So the basic idea of soft information relaying is to benefit from coding gain while still transmitting reliability information.

For the uncoded case with BPSK modulation, the optimal relay function in terms of bit error rate (BER) was derived in [2]. The solution requiring the LambertW function normalized to the power constraint is quite complex to implement. In [3] the optimal relay function considering the mean-squared-error (MSE) was derived analytically and was called Estimate-Forward (EF). Furthermore, EF was numerically shown in [4] to be also optimal in terms of capacity. The resulting relay function corresponds to the conditioned expectation value of the transmitted symbols. The signal to be forwarded can be calculated easily in contrast to the result of [2] as the normalization to the relay power is independent of the argument of the relay function. Furthermore, EF can be extended to higher order modulations as, e.g., 16-QAM quite easily.

The first relay function using soft information exploiting coding gain in coded systems was achieved for BPSK by transmitting log-likelihood-ratios (LLRs) for the code bits normalized to the power constraint. This so-called Decode-Amplify-Forward (DAF) was applied in several publications, e.g., in [5], [6], [7], [8]. However, DAF is not applicable to higher order modulation schemes as LLRs are only well defined for binary signals. In contrast to this, in [9] and [10] soft bits representing the expectation values of the BPSK modulated code bits after channel decoding in the relay are used. The optimality of transmitting the expectation values for BPSK in terms of MSE was extended in [11] to the coded case assuming a-posteriori probability (APP) decoding at the relay. The resulting relay function called Decode-Estimate-Forward (DEF) was shown to outperform DF in terms of MSE and BER performance. Additionally, it was shown that DAF performs much worse than DEF in the case of BPSK and tends to an error floor for high SNR. In contrast to DAF, the extension of DEF to higher order modulation is possible and will be shown to outperform AF and DF for a large variety of scenarios.

The paper is organized as follows: The basic system model is introduced in Section II and is assumed in Section III to derive the considered relay functions. In Section IV-A the performance of the proposed approach for the basic system is evaluated by simulations. Furthermore, in Section IV-B a more general system setup is introduced for further simulations and the results are discusses in detail. Section V gives a conclusion of this work.

II. System Description

The basic setup is shown in Figure 1 with one source S and one destination D communicating by the help of one relay R. The information bit vector[1] $\mathbf{b} = (b_1, b_2, ..., b_{N_u})$ of length N_u is encoded at the source with a channel code $\mathcal{C}$. The interleaved

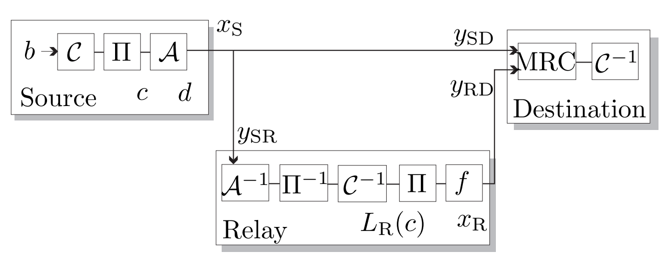

Fig. 1. Block diagram of the basic relay network setup applying channel coding and a general relay function f

[1]Throughout the paper vectors are denoted as bold letters and i-th element as italic letter, e.g. $\mathbf{b}$ and b_i. For arbitrary elements the subscript is omitted. Soft estimates of bits are identified with a tilde $\tilde{b}$ and hard estimates by $\hat{b}$.

code bits c are mapped to symbols d of the alphabet $\mathcal{A}$ with cardinality $|\mathcal{A}| = M$. These symbols are normalized to the source transmit power P_{S} yielding $\mathbf{x}_{\mathrm{S}} = \sqrt{P_{\mathrm{S}}} \cdot \mathbf{d}$. The received signals at the relay $\mathbf{y}_{\mathrm{SR}}$ and the destination $\mathbf{y}_{\mathrm{SD}}$ is then given by

$$\mathbf{y}_{\mathrm{SR}} = h_{\mathrm{SR}} \mathbf{x}_{\mathrm{S}} + \mathbf{n}_{\mathrm{SR}} \quad \text{and} \quad \mathbf{y}_{\mathrm{SD}} = h_{\mathrm{SD}} \mathbf{x}_{\mathrm{S}} + \mathbf{n}_{\mathrm{SD}} \ , \tag{1}$$

with h_{SR}, h_{SD}, $\mathbf{n}_{\mathrm{SR}}$ and $\mathbf{n}_{\mathrm{SD}}$ denoting the channel coefficients and noise vectors of the source-relay (SR) and the source-destination (SD) channel, respectively. We will consider AWGN channels as well as quasi-static Rayleigh fading for the simulations. For AWGN channels the channel coefficients include the path loss determined by the distances, e.g., l_{SR} and the path loss exponent which is assumed to be 3

$$|h_{\mathrm{SR}}|^2 = l_{\mathrm{SR}}^{-3} \ . \tag{2}$$

In the case of block Rayleigh fading channels the channel coefficients are Rayleigh distributed random variables with

$$\mathrm{E}\left\{|h_{\mathrm{SR}}|^2\right\} = l_{\mathrm{SR}}^{-3} \ . \tag{3}$$

It is always assumed that all entities transmit in different time slots. This allows to neglect interference between different links and keeps the derivations simple.

III. Relaying Functions

In the case of classical Decode-Forward, the received signal is decoded at the relay and the hard symbol decisions $\hat{d}_{\mathrm{R}}$ are forwarded to the destination

$$\mathbf{x}_{\mathrm{R}}^{\mathrm{DF}} = \sqrt{P_{\mathrm{R}}} \cdot \hat{\mathbf{d}}_{\mathrm{R}} \ , \tag{4}$$

where P_{R} denotes the power constraint of the relay. If no error detection scheme is applied, this approach suffers from error propagation, as even erroneously decoded massages are forwarded to the destination. In contrast to this, Amplify-Forward simply scales the received signal to the relay power constraint and forwards this analog signal to the destination. In this case the constellation constraints and the redundancy due to the channel code are not exploited at the relay.

$$\mathbf{x}_{\mathrm{R}}^{\mathrm{AF}} = \frac{\sqrt{P_{\mathrm{R}}}}{\sqrt{\mathrm{E}\left\{|\mathbf{y}_{\mathrm{SR}}|^2\right\}}} \cdot \mathbf{y}_{\mathrm{SR}} \ . \tag{5}$$

If a soft-input-soft-output channel decoder is applied at the relay, reliability information about the information and code bits are available. In the case of an APP decoder, the resulting soft information about the code bits can be described by LLRs

$$L_{\mathrm{R}}(c) \triangleq L(c|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}) = \log\left(\frac{p(c=0|\mathbf{y}_{\mathrm{SR}}, \mathcal{C})}{p(c=1|\mathbf{y}_{\mathrm{SR}}, \mathcal{C})}\right) \ . \tag{6}$$

If BPSK is considered, these LLRs can simply be scaled to the transmit power constraint and then forwarded to the destination [5]. However, this approach DAF is not applicable to higher order modulation schemes as LLRs about transmit symbols are not defined properly for QAM but only for binary symbol alphabets.

Estimate-Forward (EF) was derived in [3] for an uncoded system with the aim to minimize the residual error between received signals at the destination and the transmitted symbols. In [11] this approach was extended to the coded case for BPSK and was shown to clearly outperform DAF. As already stated in [3], the EF relay function minimizes the MSE independently of the signal constellation. Therefore in this paper we will generalize this function to the coded case with arbitrary signal constellations. For the purpose of minimal MSE, the source-relay channel and the relay function itself are modeled as one superchannel as depicted in Figure 2. The equivalent noise η on this superchannel is defined to be the uncorrelated error between the transmitted symbols d and the corresponding estimates at the relay denoted by $\tilde{d}_{\mathrm{R}}$. For AF the values of

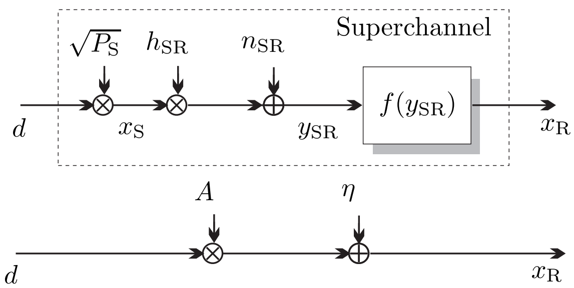

Fig. 2. Definition of a superchannel including the source-relay channel and the relay function f

A and σ_η^2 only depend on the variance of n_{SR} and on the normalization factor at the relay; for DF only the SNR on the last hop is taken into account, as the receiver assumes error-free decoding at the relays. Minimizing the noise variance of the overall channel including the relay-destination channel is equivalent to minimizing the noise variance of the superchannel σ_η^2 yielding the minimum mean-squared-uncorrelated-error (MSUE) and the corresponding relay function

$$x_{\mathrm{R}} = \frac{\sqrt{P_{\mathrm{R}}}}{\sqrt{\mathrm{E}\left\{|\tilde{d}_{\mathrm{R}}|^2\right\}}} \cdot \tilde{d}_{\mathrm{R}} \tag{7}$$

is optimal in terms of MSUE at the receiver. For memoryless channels as uncoded AWGN, one element $\tilde{d}_{\mathrm{R}}$ depends only on one received symbol $\tilde{d}_{\mathrm{R}} = f\left(y_{\mathrm{SR}}\right)$. But in a coded system the relay can make use of the channel code, and as a consequence, the channel is not memoryless and the whole receive vector and the code constraint have to be considered. In this case the relay function can be expressed more generally as

$$x_{\mathrm{R}} = f\left(\mathbf{y}_{\mathrm{SR}}, \mathcal{C}\right) \ . \tag{8}$$

As shown for uncoded transmission in [3], the conditional expectation of the transmitted bits minimizes the MSUE at the destination. As the only difference between MSE and MSUE is a scaling factor we will focus on the MSE due to simpler derivations

$$\mathrm{MSE} = \mathrm{E}\left\{\left(\tilde{d}_{\mathrm{R}} - d\right)^2 |\mathbf{y}_{\mathrm{SR}}, \mathcal{C}\right\} \ . \tag{9}$$

The relay function yielding the minimum MSE can be found by setting the derivative of (9) to zero

$$\frac{\partial \mathrm{MSE}}{\partial \tilde{d}_{\mathrm{R}}} = 2\mathrm{E}\left\{\left(\tilde{d}_{\mathrm{R}} - d\right)|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}\right\} \stackrel{!}{=} 0 \tag{10}$$

leading to

$$\tilde{d}_{\mathrm{R}} = \mathrm{E}\left\{d|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}\right\} \ . \tag{11}$$

The resulting estimated symbol $\tilde{d}_{\mathrm{R}}$ is then normalized to the power constraint of the relay (7). In the special case of BPSK, this conditional expectation corresponds to the well-known soft bits $\tilde{d}_{\mathrm{R}}^{\mathrm{DEF}} = \mathrm{E}\left\{d|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}\right\} = \tanh\left(L_{\mathrm{R}}(c)/2\right)$. This result justifies the usage of soft bits for relaying in [9].
To extend the DEF relay function to higher order modulation schemes, we consider the general definition of the expectation value given by the sum of all symbols of $\mathcal{A}$ weighted by the corresponding symbol probability

$$\mathrm{E}\{d|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}\} = \sum_{d\in\mathcal{A}} d \cdot p(d|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}) \ . \tag{12}$$

Symbol probabilities based on the product of APP probabilities delivered by the channel decoder are suboptimal, because bits corresponding to one symbol are not independent. To calculate the conditional probability $p(d|\mathbf{y}_{\mathrm{SR}}, \mathcal{C})$ correctly, we have to split it up into an intrinsic and an extrinsic part

$$p(d|\mathbf{y}_{\mathrm{SR}}, \mathcal{C}) \sim p(y_{\mathrm{SR}}|d) \cdot p(d|\mathbf{y}_{\mathrm{SR}\backslash y}, \mathcal{C}) \ . \tag{13}$$

The probability $p(d|\mathbf{y}_{\mathrm{SR}\backslash y}, \mathcal{C})$ denotes the extrinsic probability delivered by the channel decoder about d. It can be calculated as the product of the corresponding extrinsic code bit probabilities as these probabilities can be assumed to be independent of each other

$$p(d|\mathbf{y}_{\mathrm{SR}\backslash y}, \mathcal{C}) = \prod_{i=1}^{\mathrm{ld}(M)} p(c_i(d)|\mathbf{y}_{\mathrm{SR}\backslash y}, \mathcal{C}) \ . \tag{14}$$

Here, $p(c_i(d)|\mathbf{y}_{\mathrm{SR}\backslash y}, \mathcal{C})$ denotes the extrinsic probability of the i-th bit of symbol d conditioned on the received vector and the code constraint. The intrinsic part $p(y_{\mathrm{SR}}|d)$ in (13) is the probability based on the channel observation and can be expressed as

$$p(y_{\mathrm{SR}}|d) \sim \exp\left(-\frac{|y_{\mathrm{SR}} - d|^2}{2\sigma_n^2}\right) \tag{15}$$

for Gaussian noise. At the destination, soft-output demappers are applied to all arriving signals assuming the equivalent channel coefficients A and the variance of the equivalent noise σ_η^2 as introduced in Figure 2. Although the overall disturbance is not Gaussian distributed anymore, it is common to assume a Gaussian channel at the receiver to calculate the LLRs, as an exact calculation of the LLRs by the demappers would be a hard task, especially for higher order modulation schemes. For BPSK the loss due to this simplification is shown in [11] to be quite small. The resulting LLR values are simply summed up to get the overall LLR which is in fact Maximum-Ration-Combining (MRC).
It should be mentioned that the parameters A and σ_η^2 have to be estimated in a real system, but the aim of channel and SNR estimation is beyond the scope of this paper so we assume perfect knowledge of these parameters.

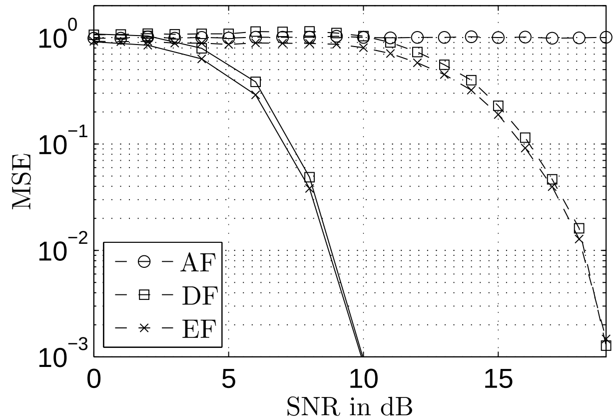

Fig. 3. Mean squared error of relay functions for 16-QAM in an uncoded system (dashed) and convolutionally coded system (solid)

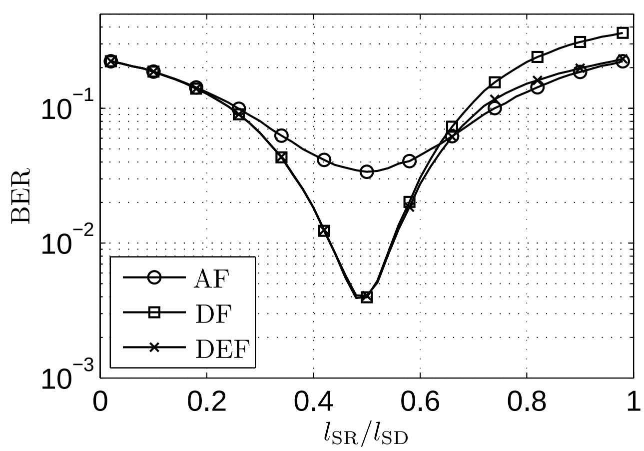

Fig. 4. BER with 16-QAM for different positions of the relay with [5,7] convolutional code, $N_u = 200$, AWGN channels, $\mathrm{SNR}_{\mathrm{SD}} = 0\,\mathrm{dB}$

IV. Performance Evaluation

A. Basic Setup

In Figure 3 the MSE (9) at the output of the relay for 16-QAM is depicted for the uncoded case as well as for a system applying the $[5,7]_o$ convolutional code. In addition the results for AF are shown and it can be observed, that EF for uncoded and DEF for coded systems outperform the corresponding hard decision forwarding as well as AF. This result illustrates the combination of the benefits of both approaches. As our aim was to minimize the MSE this result is not surprising, but in the following figures the bit error rate (BER) is plotted for the different relay schemes. In Figure 4 a system with one relay located between source and destination is considered. The BER of AF, DF and DEF are plotted over the position of the relay. First we can observe the well known property that AF performs well if the relay is near the destination whereas

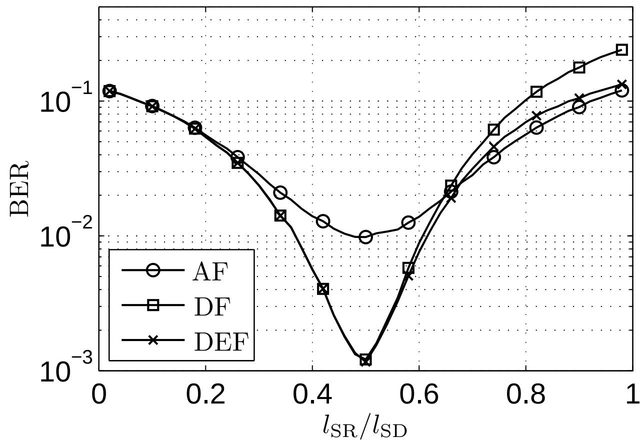

Fig. 5. BER with 64-QAM for different positions of the relay with [5,7] convolutional code, $N_u = 200$, AWGN channels, $\mathrm{SNR}_{\mathrm{SD}} = 6\,\mathrm{dB}$

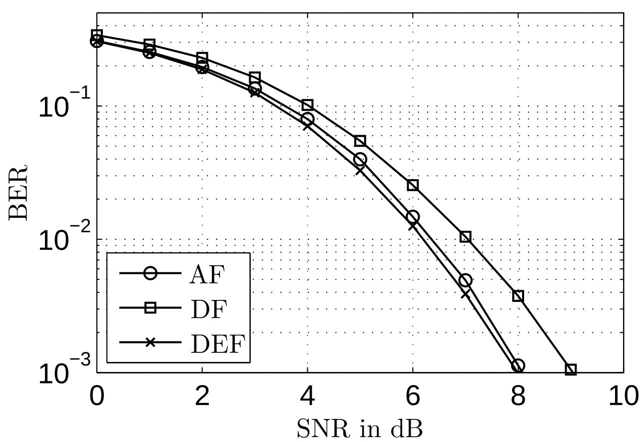

Fig. 6. BER of relay functions with channel coding for 16-QAM in a symmetric system with AWGN channels, $N_u = 200$

DF performs better in the other case. But DEF performs almost as good as the best choice in all cases, i.e., on the one hand DEF outperforms AF up to $l_{\mathrm{SR}}/l_{\mathrm{SD}} = 0.65$ and on the other hand outperforms DF for $l_{\mathrm{SR}}/l_{\mathrm{SD}} > 0.6$. So DEF is a good compromise between AF and DF.

It should be mentioned that AF outperforms DEF for low SNR on the first hop in terms of BER although DEF is best in terms of MSE. The reason of this discrepancy is that the BER does not only depend on the variance of the error but on the entire error distribution. Furthermore, the exact distribution of η is not taken into account by the demapper. Nevertheless, the MSE criterion provides a good hint on the BER performance while making the derivations simpler. To show the feasibility of DEF to arbitrary signal constellations like M-QAM Figure 5 shows the results for the same system setup with 64-QAM. As the results look quite similar, we will focus on 16-QAM in the sequel.

The results for a slightly different setup are shown in Figure 6. A symmetric system is considered, i.e., the distances and therefore the SNRs of all transceivers are the same $l_{\mathrm{SR}} = l_{\mathrm{SD}} = l_{\mathrm{RD}}$. In this case DEF outperforms DF and AF over the

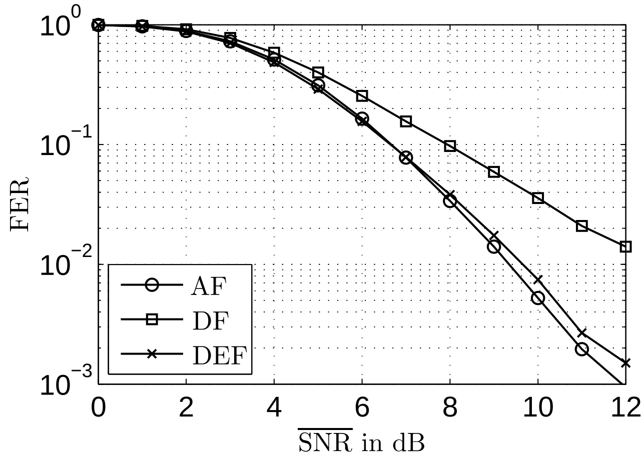

Fig. 7. BER of relay functions with channel coding for 16-QAM in a symmetric system with Rayleigh fading channels, $N_u = 200$

whole SNR range. We only considered AWGN channels so far as defined in (2) but a more realistic scenario is modeled by Rayleigh fading (3). The frame error rates (FER) for a symmetric system with block Rayleigh fading are shown in Figure 7. For coded systems with block fading channels FER is a more meaningful performance measure than BER. The main effect which can be observed here is the bad performance of DF caused by the maximum-ratio-combining at the destination. As mentioned before, the destination assumes that all bits forwarded by the relay are correct and the SNR used for the MRC only takes the last hop into account. The larger the difference between the instantaneous SNR becomes, the larger becomes the mismatch of the MRC degrading the overall performance. AF and DEF perform much better in this case as both take the quality of the first hop also into account.

B. Multihop- and Parallel Transmission

To emphasize the good behavior of DEF in different scenarios, the two system setups shown in Figure 8 corresponding to a multihop transmission with 3 relays and a parallel constellation are considered in the sequel. In both cases no direct link between source and destination is assumed. All

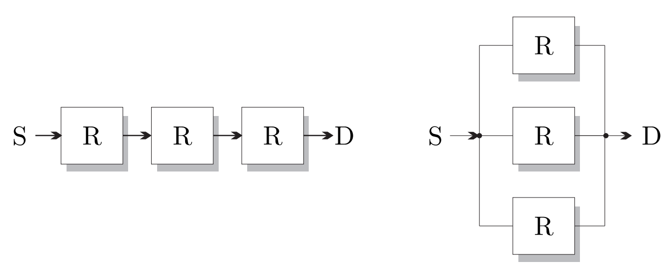

Fig. 8. Block diagram of multihop and parallel relay network

links are assumed to have the same average SNR in order to keep the number of variables tractable. First, AWGN channels are assumed for these two scenarios. In the parallel case DEF performs best, but DF and AF do not loose much. The more interesting, but nevertheless not surprising result, is obtained for a serial configuration of the 3 relays. AF performs very bad in this case whereas DF and DEF behave similar. This is quite clear as for AF all noise processes are summed up with certain normalization factors and the SNR gets worse with each additional hop. DF and DEF can reduce the effect of noise of the previous hops due to the coding gain exploited at the relays. The situation changes drastically if block Rayleigh

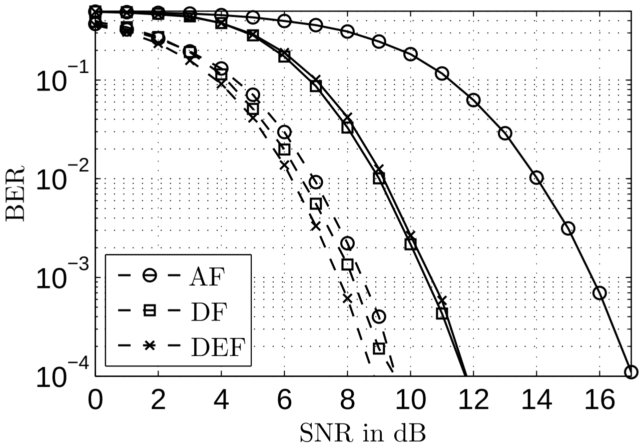

Fig. 9. BER for different relay functions with channel coding for 16-QAM, AWGN channels and 3 relays in serial (solid) and in parallel (dashed), $N_u = 200$

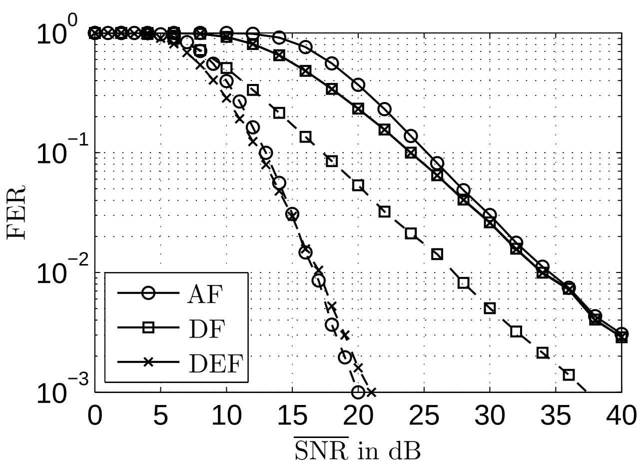

Fig. 10. BER for different relay functions with channel coding for 16-QAM, Rayleigh fading channels and 3 relays in serial (solid) and in parallel (dashed), $N_u = 200$

fading channels are assumed in Figure 10. For the serial case the differences of the schemes are relatively small; even AF does not show a large loss which is in contrast to the AWGN case in Figure 9. But for the parallel case the DF scheme fails miserably due to the SNR mismatch at the MRC. A similar behavior was already observed in the symmetric network shown in Figure 7.

To sum up, AF and DF perform very different if the scenario is changed, e.g., AF performs well if several signal paths have to be combined at the destination whereas DF is superior, if the number of hops increases. But the main result is that DEF performs better or at least close to the best of AF and DF in all scenarios considered here, including AWGN and Rayleigh fading and is therefore a more robust and flexible relaying scheme as the well known AF and DF. Furthermore, it should be emphasized that in contrast to DAF [5] which is only meaningful for binary signals, the DEF scheme presented here can be applied to arbitrary signal constellations like 16-QAM or even 64-QAM.

V. Conclusions

In this paper the extension of MMSE relaying in coded systems to arbitrary signal constellations is presented. For a large variety of system setups including AWGN and Rayleigh fading and different number and constellation of relays, it was shown by simulation that this approach called Decode-Estimate-Forward (DEF) combines the advantages of the classical relay protocols Amplify-Forward and Decode-Forward. DEF performs better or at least close to the best of AF and DF in all scenarios while AF and DF heavily fail either for multihop systems or if several relays are arranged in parallel. In addition to the enhanced performance compared to the Decode-Amplify-Forward (DAF) scheme suggested in many publications, DEF additionally can be extended to arbitrary signal constellations. In summary, DEF is a very flexible, simple and powerful non-adaptive relay scheme.

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